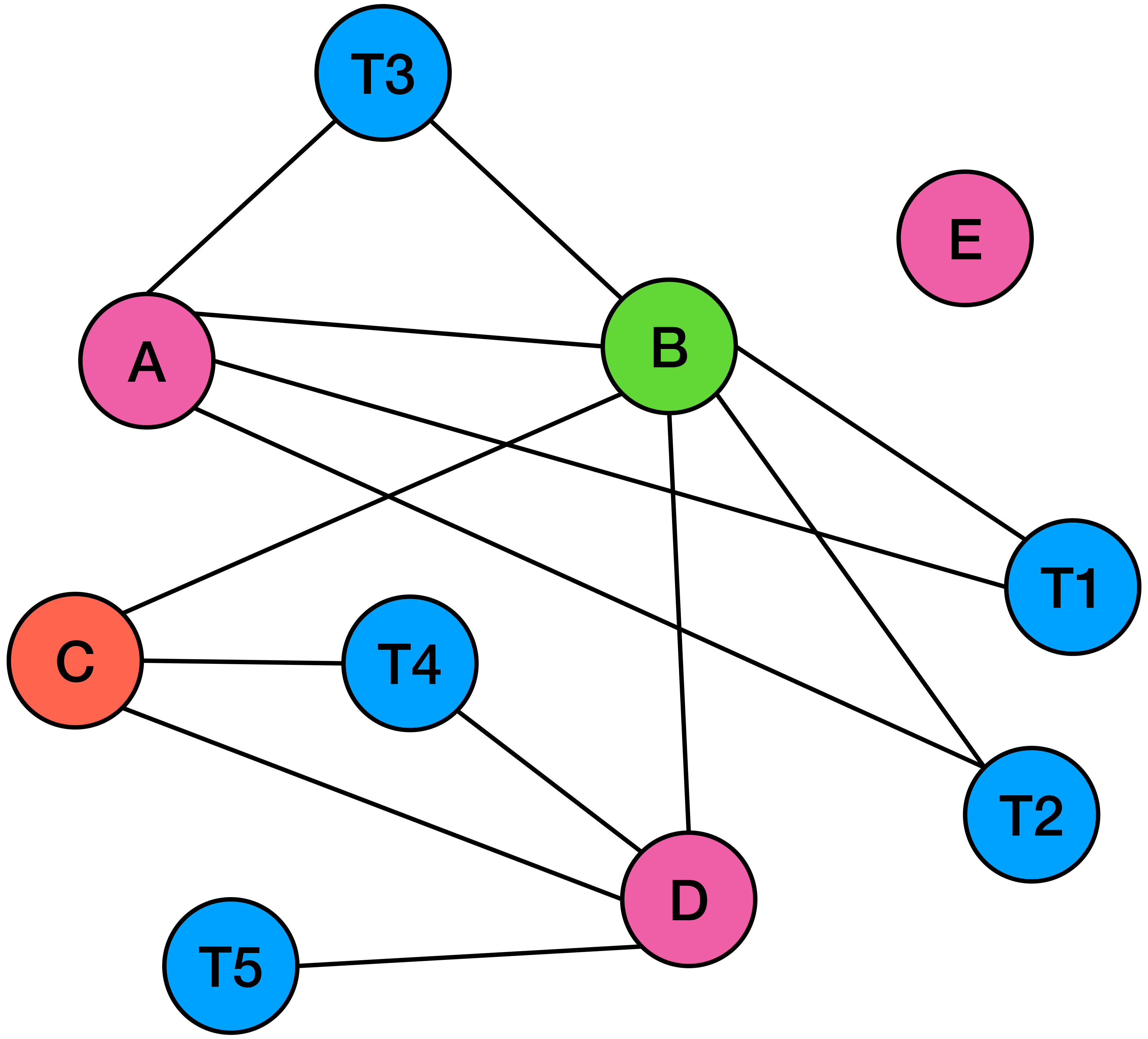
Graph Coloring Theory

graph coloring theory

- How do we color graphs?

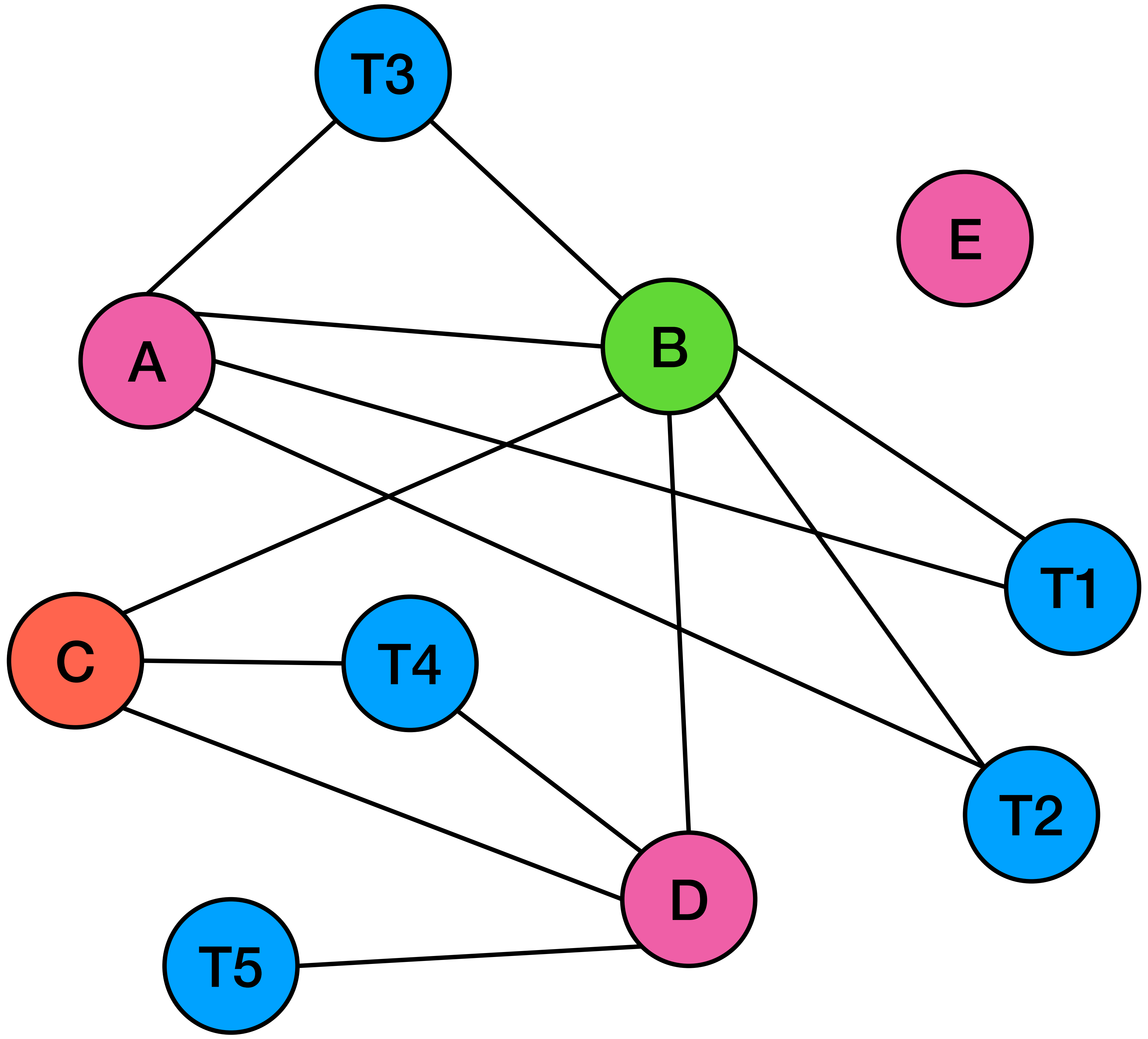



```
                     [A, B]
1:  T1 = A + B
                     [A, B, T1]
2:  T2 = A + T1
                     [A, B, T2]
3:  T3 = A + T2
                     [A, B, T3]
4:  D = A + T3
                     [B, C, D]
5:  T4 = C + B
                     [T4, C, D]
6:  T5 = T4 + C
                     [T5, D]
7:  E = T5 + D
                     [E]
```

graph coloring theory

- Can we find a coloring of a graph whenever possible?
- Can we efficiently find the optimum coloring of a graph?

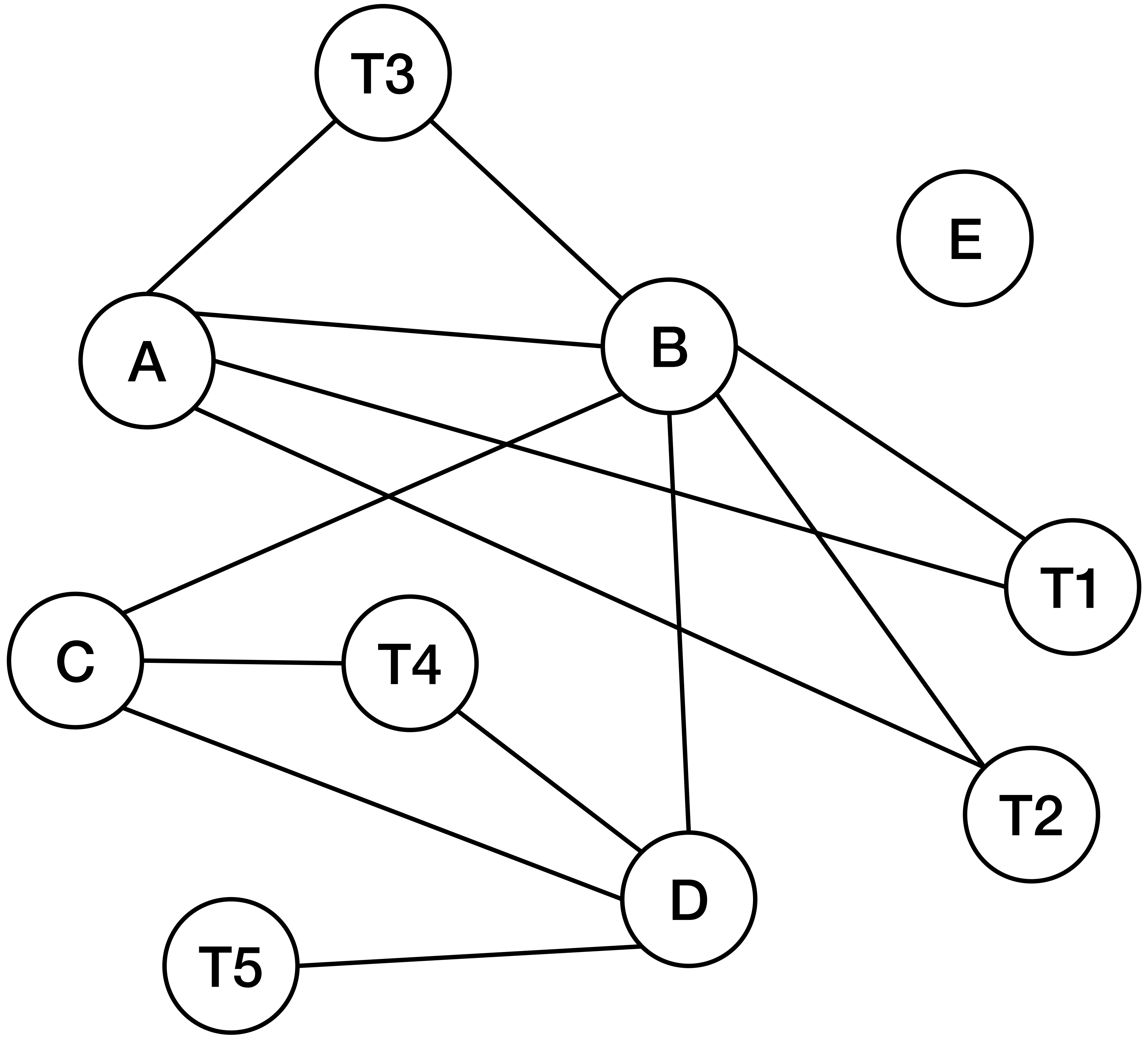
- Problem: optimal graph coloring is **NP-hard**
- (Decision problem: can a graph be colored with K or fewer colors?)



kempe's algorithm

- Algorithm from 1879 for finding a K-coloring of a graph
- Step 1: **Simplify**
  - Find a node with at most K-1 edges and remove it from the graph
  - Remember this node on the stack

- Observation: if smaller graph can be colored, bigger graph can be colored too (why?)



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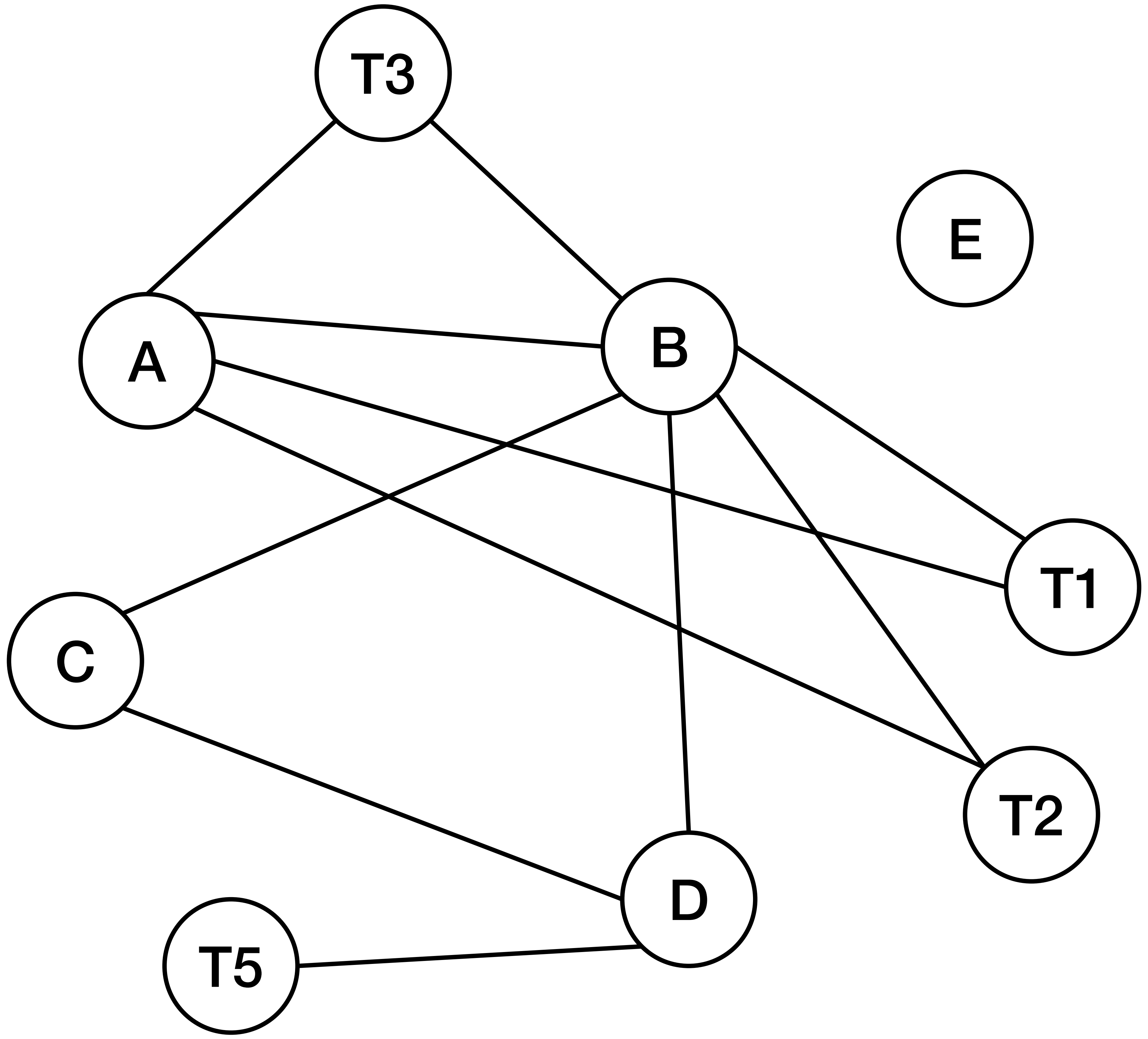

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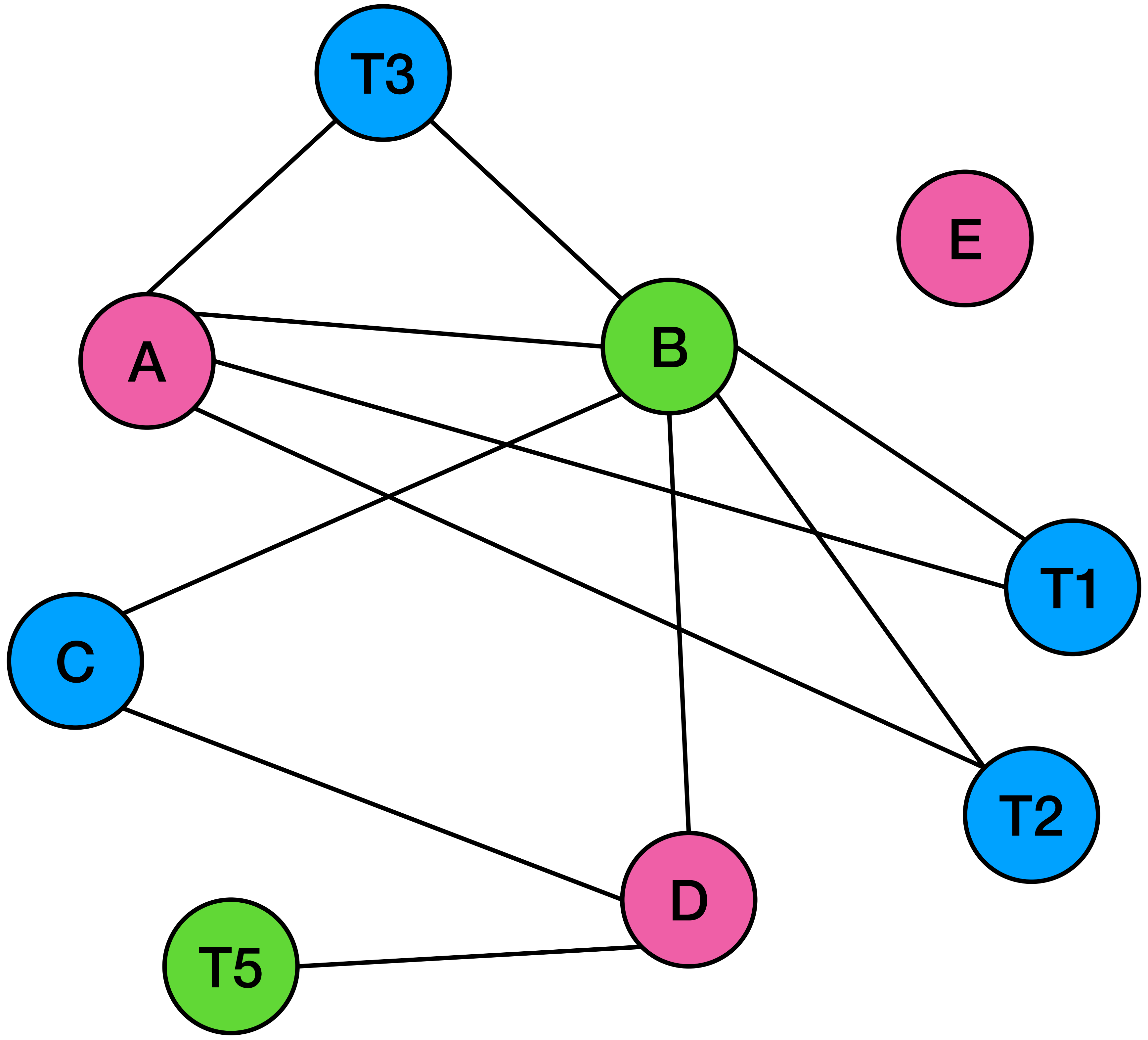
- Algorithm from 1879 for finding a K-coloring of a graph
- Step 2: **Color**
  - Once smaller graph has been colored, add node back in
  - Assign a color



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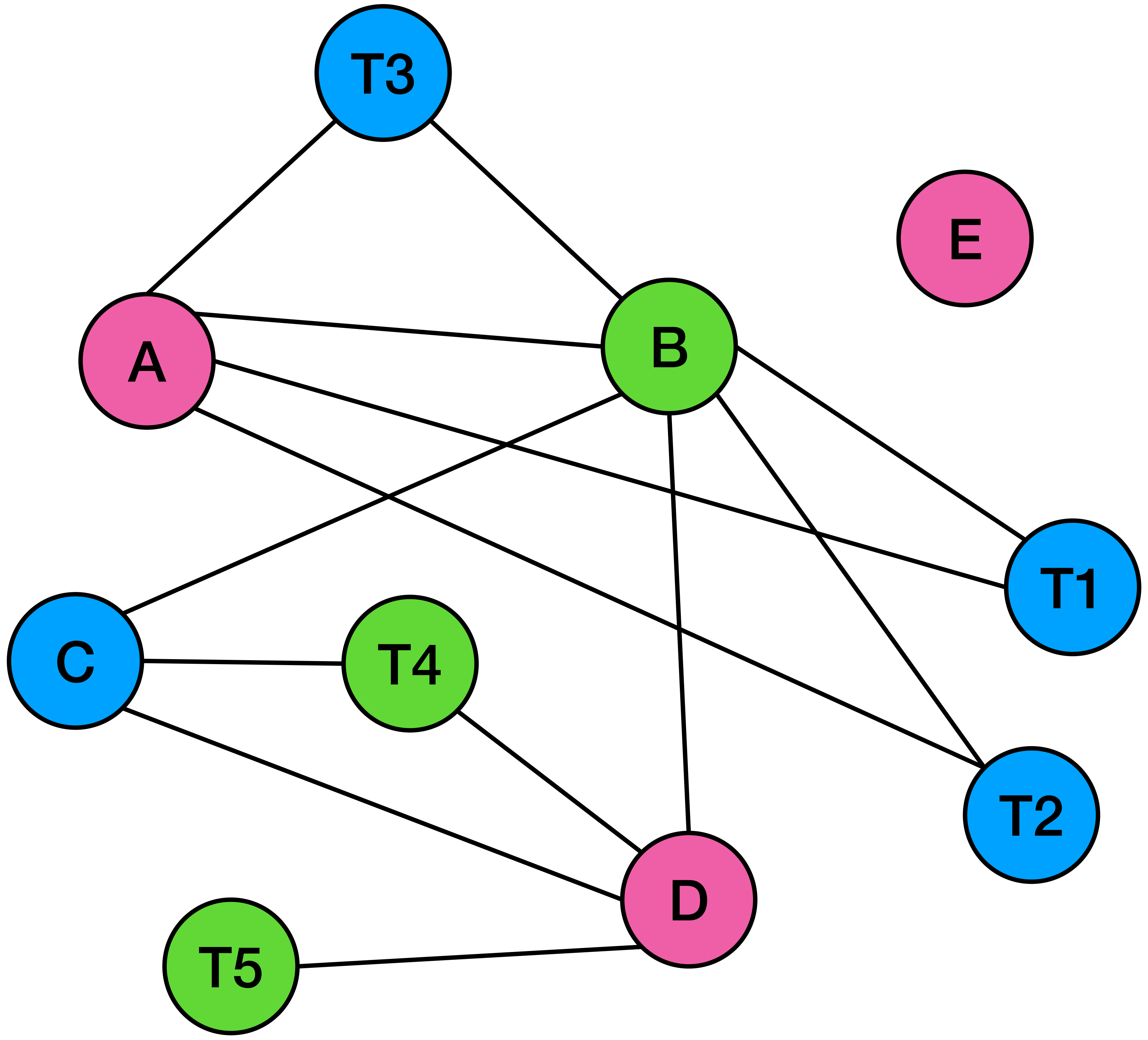

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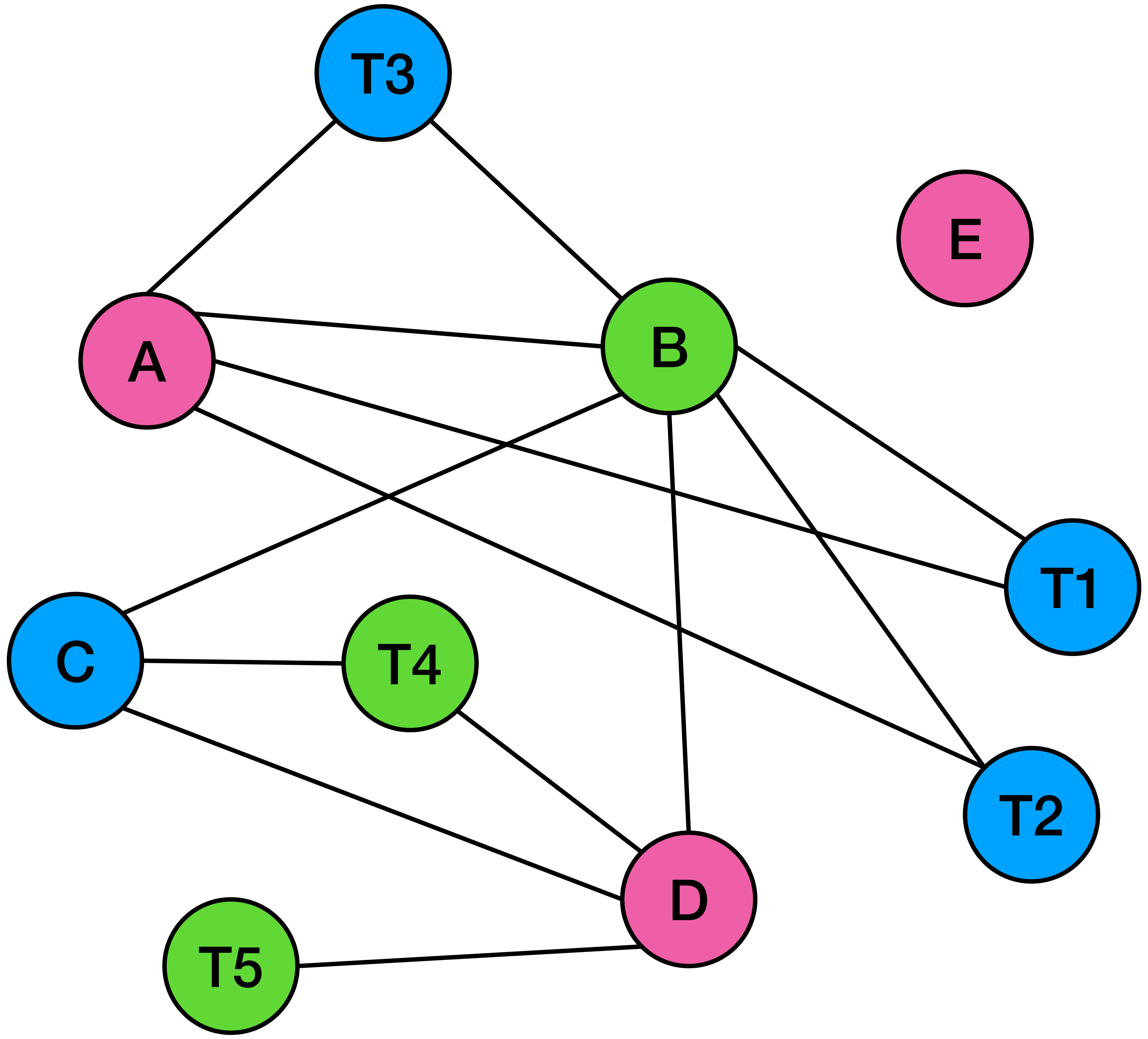
- Algorithm from 1879 for finding a K-coloring of a graph
- Apply steps 1 and 2 recursively:
  - Reduce graph
  - Color reduced graph if fewer than K vertices
  - Add nodes back into graph in reverse order they were removed



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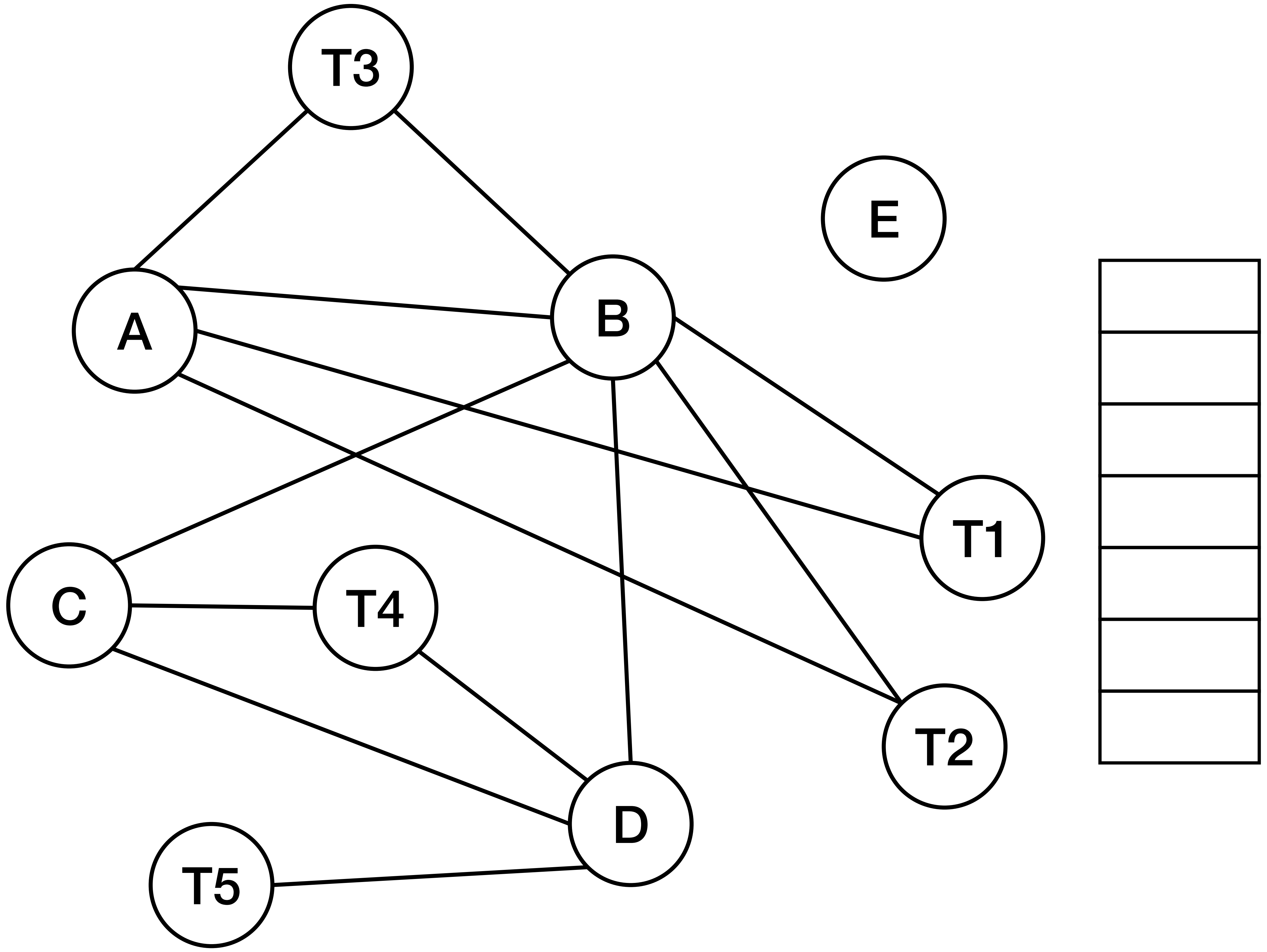

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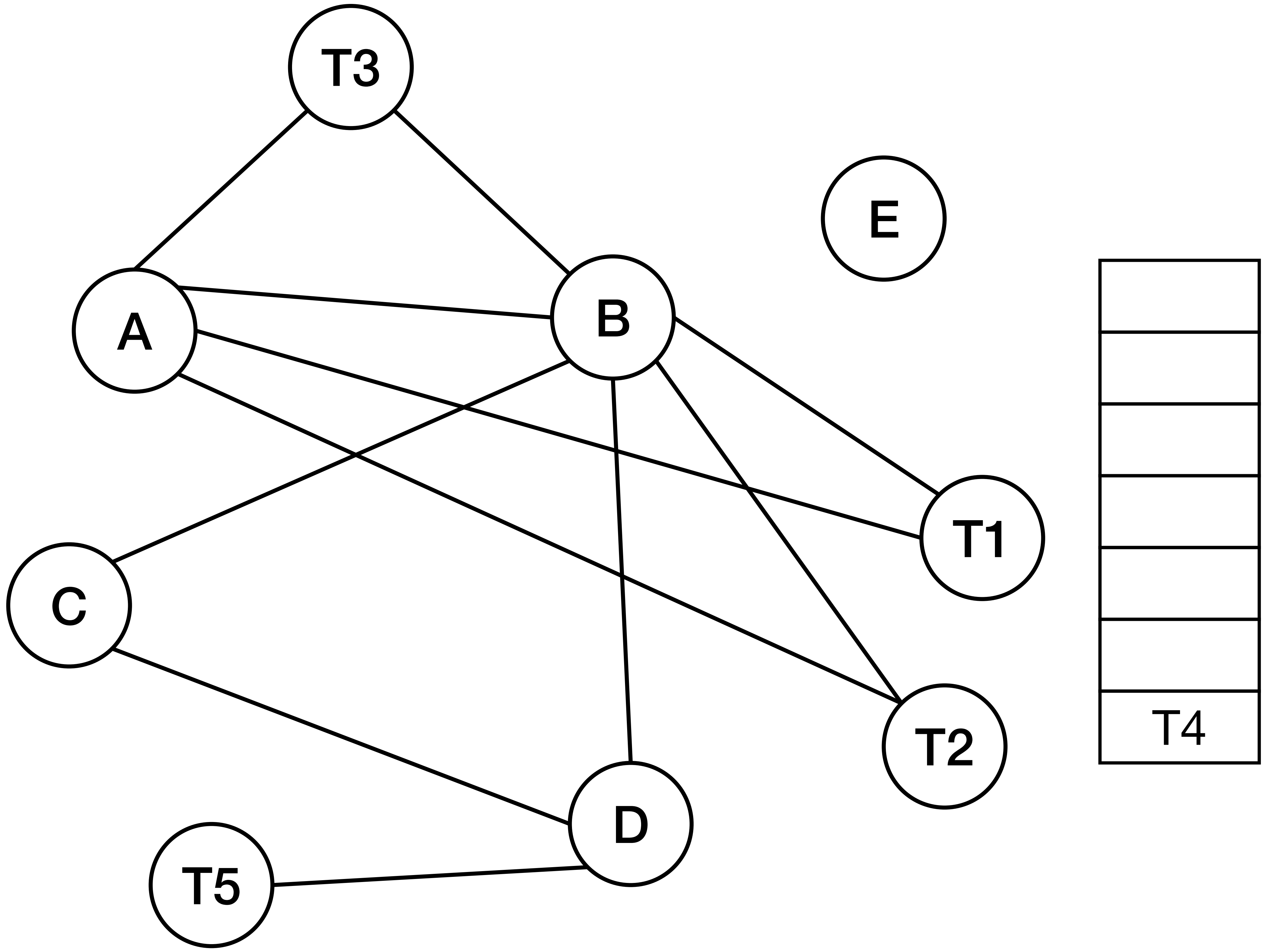

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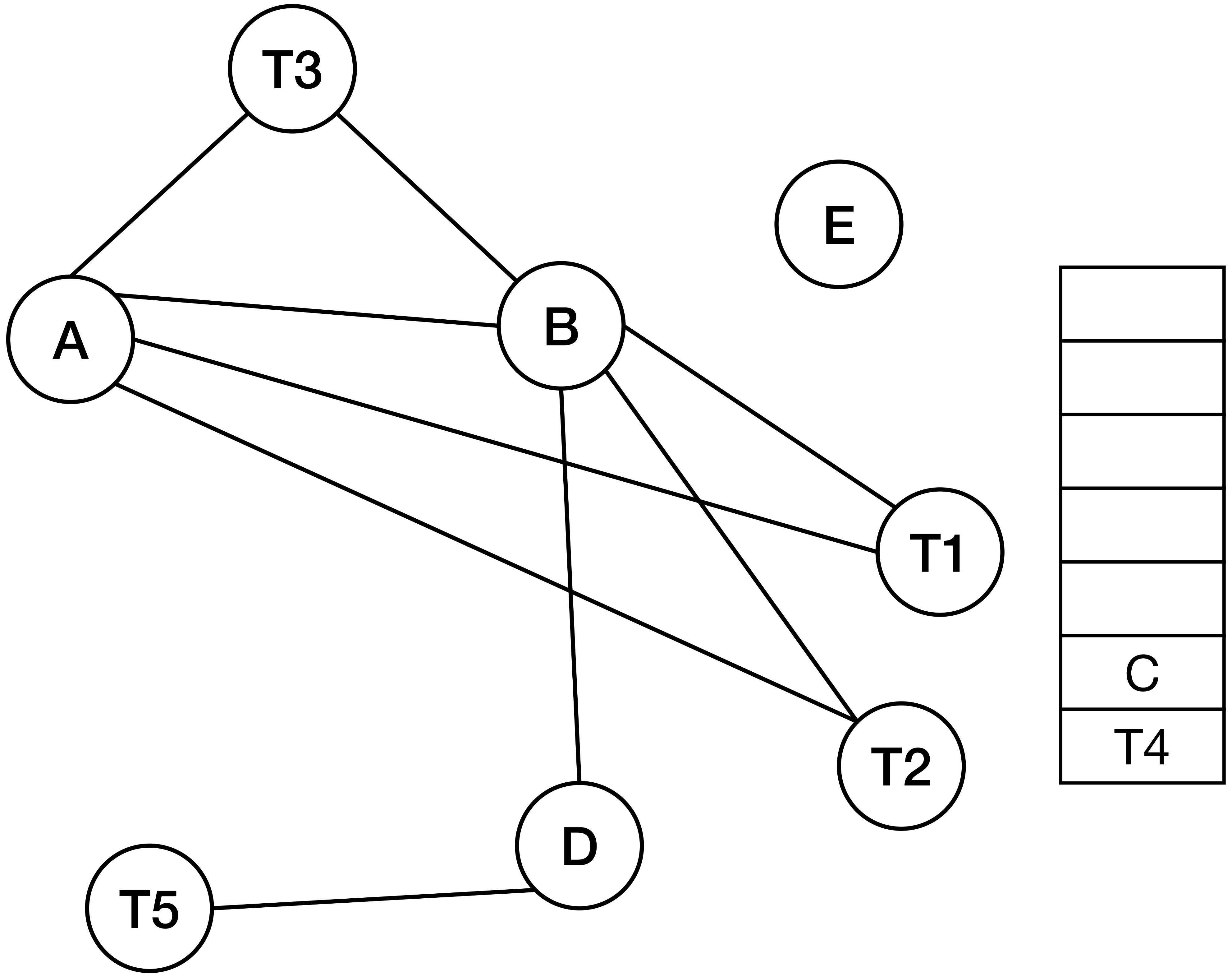

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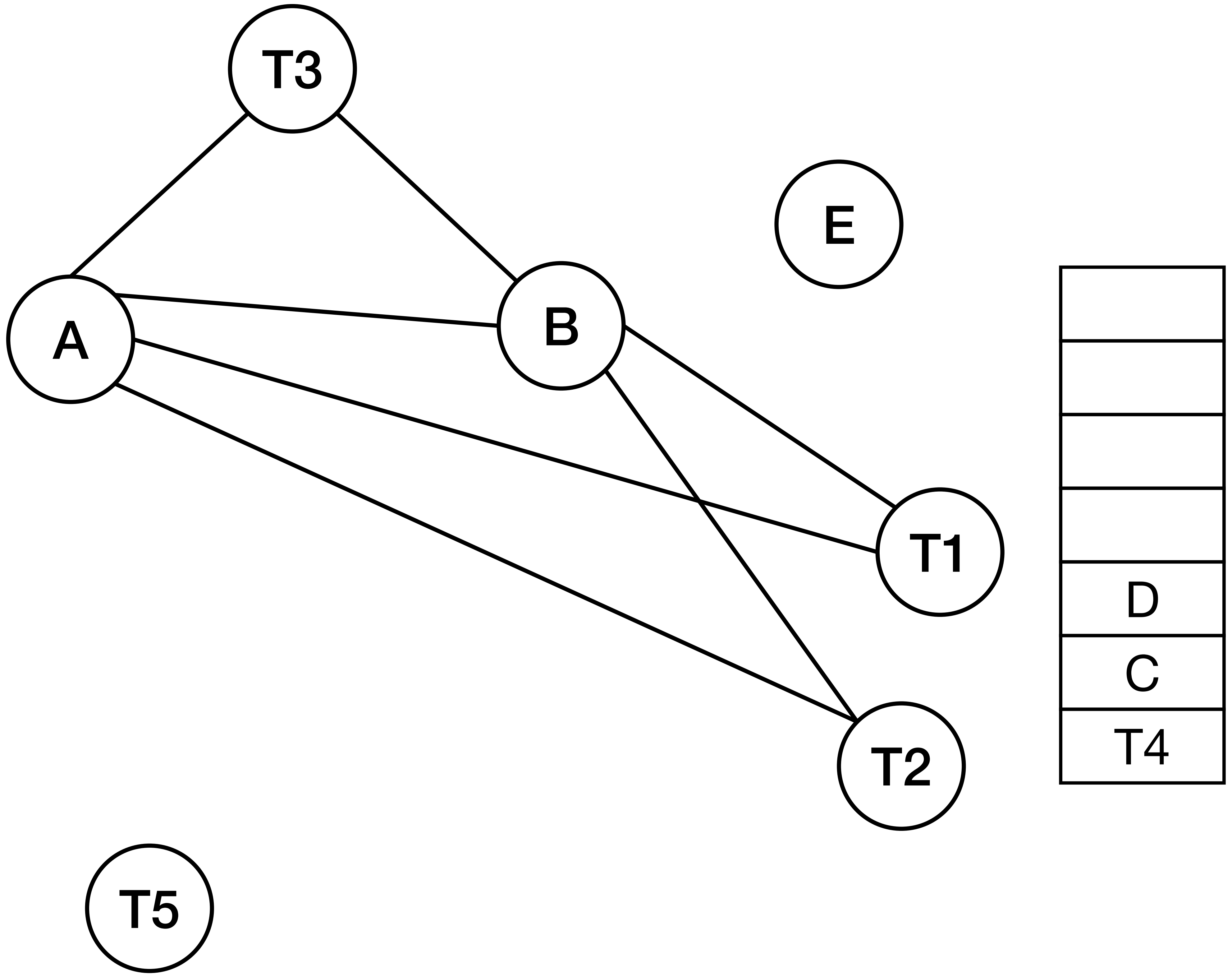

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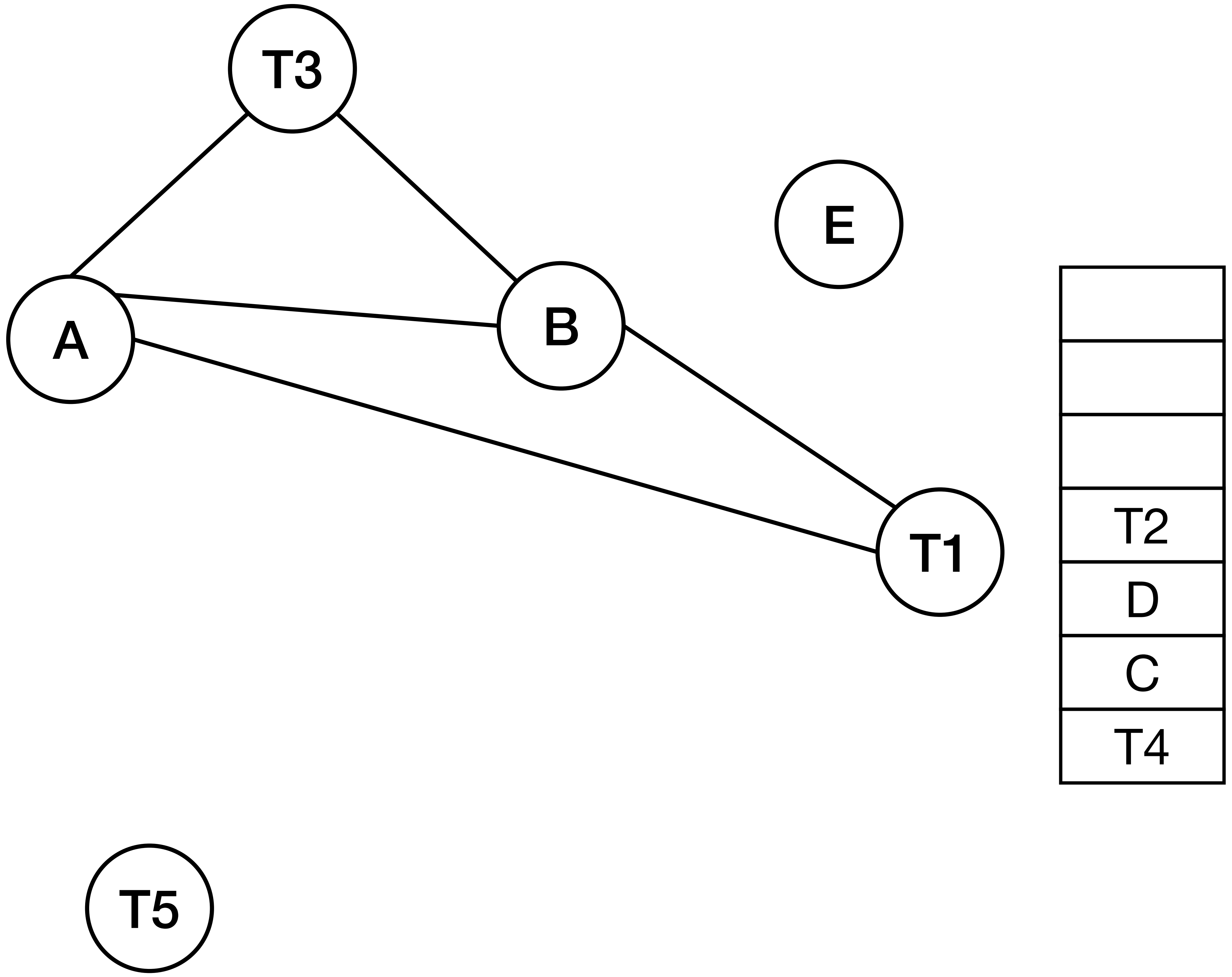

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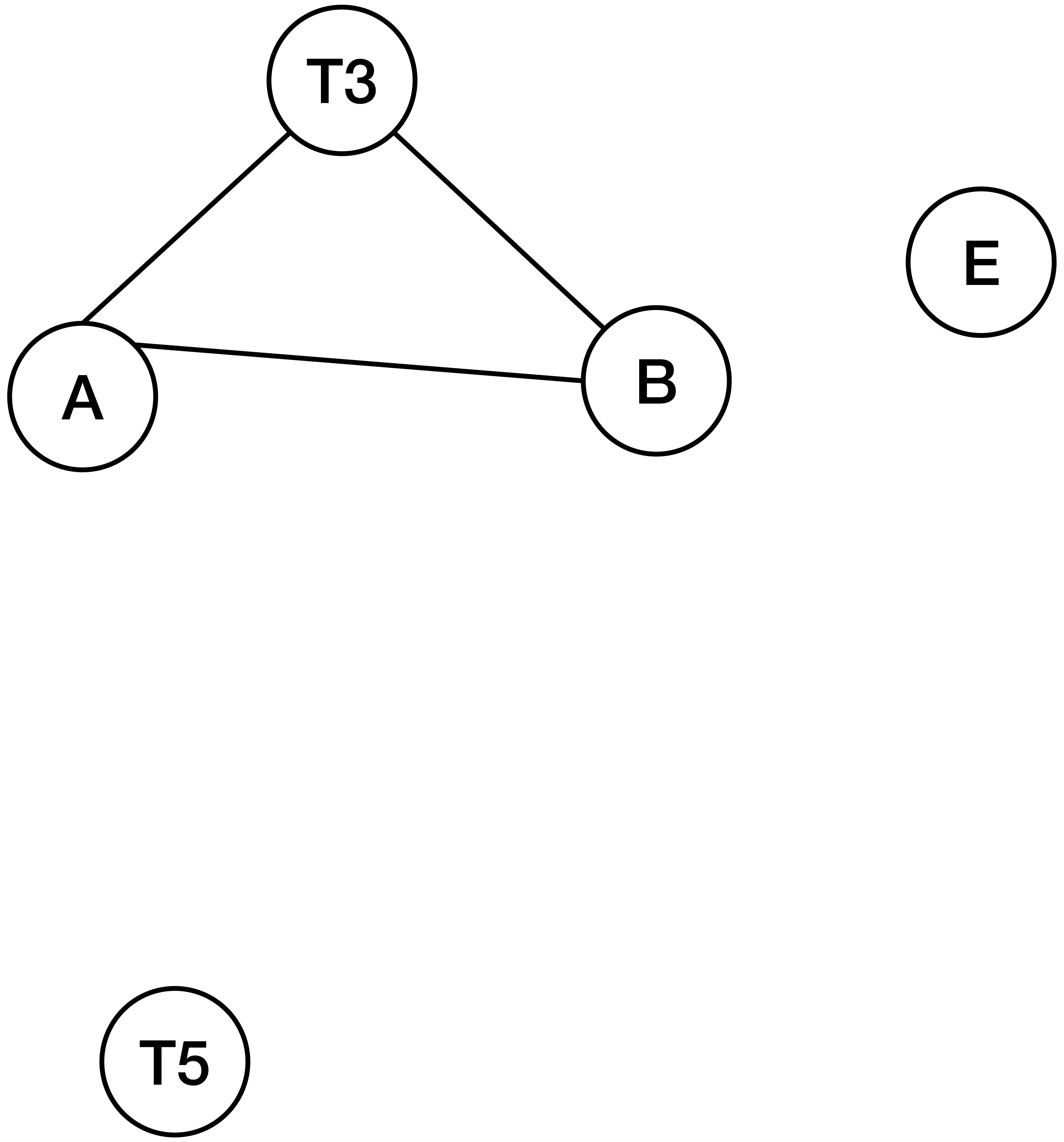

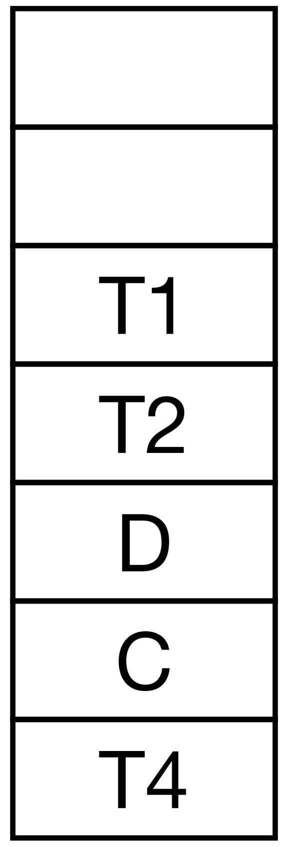

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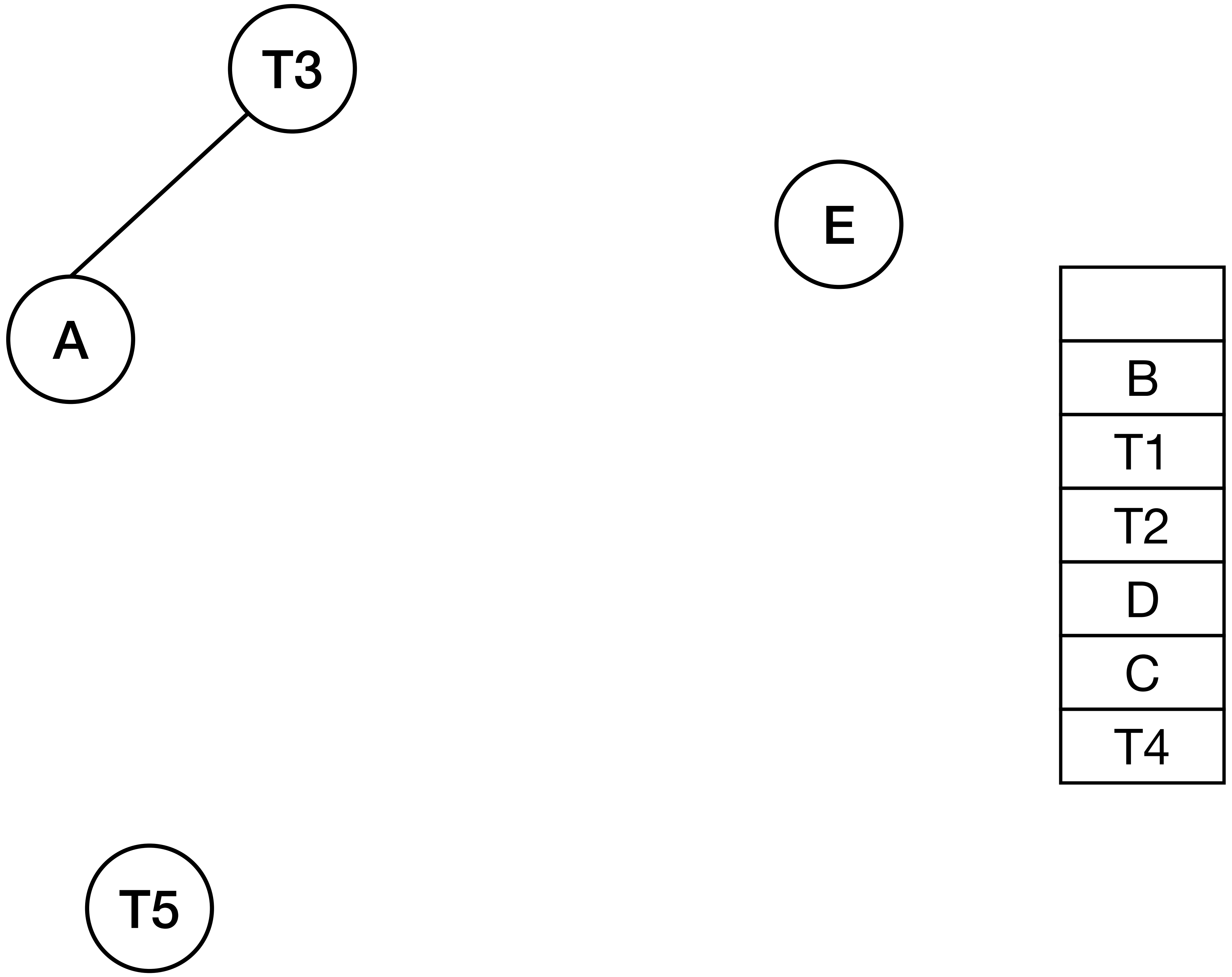

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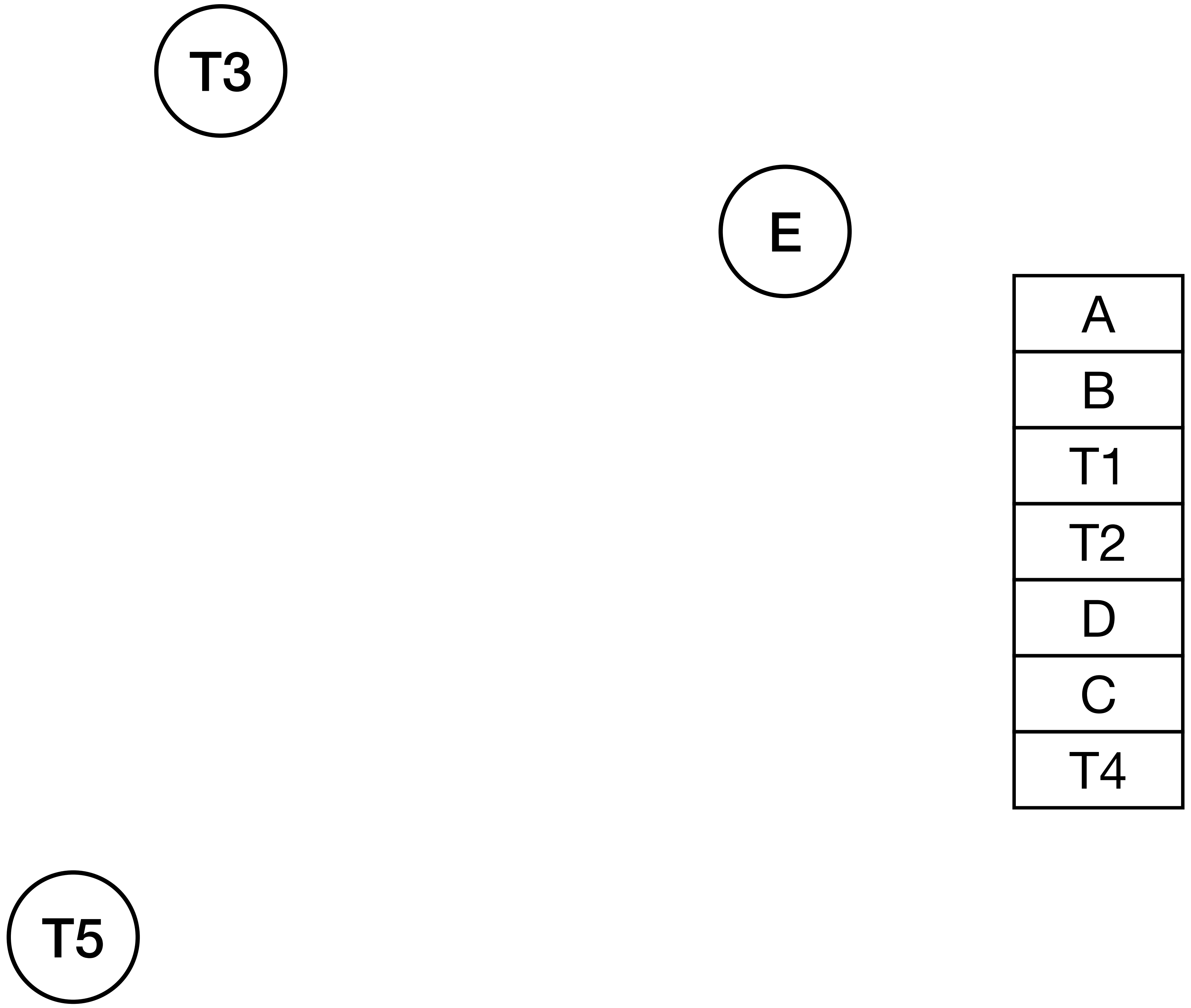

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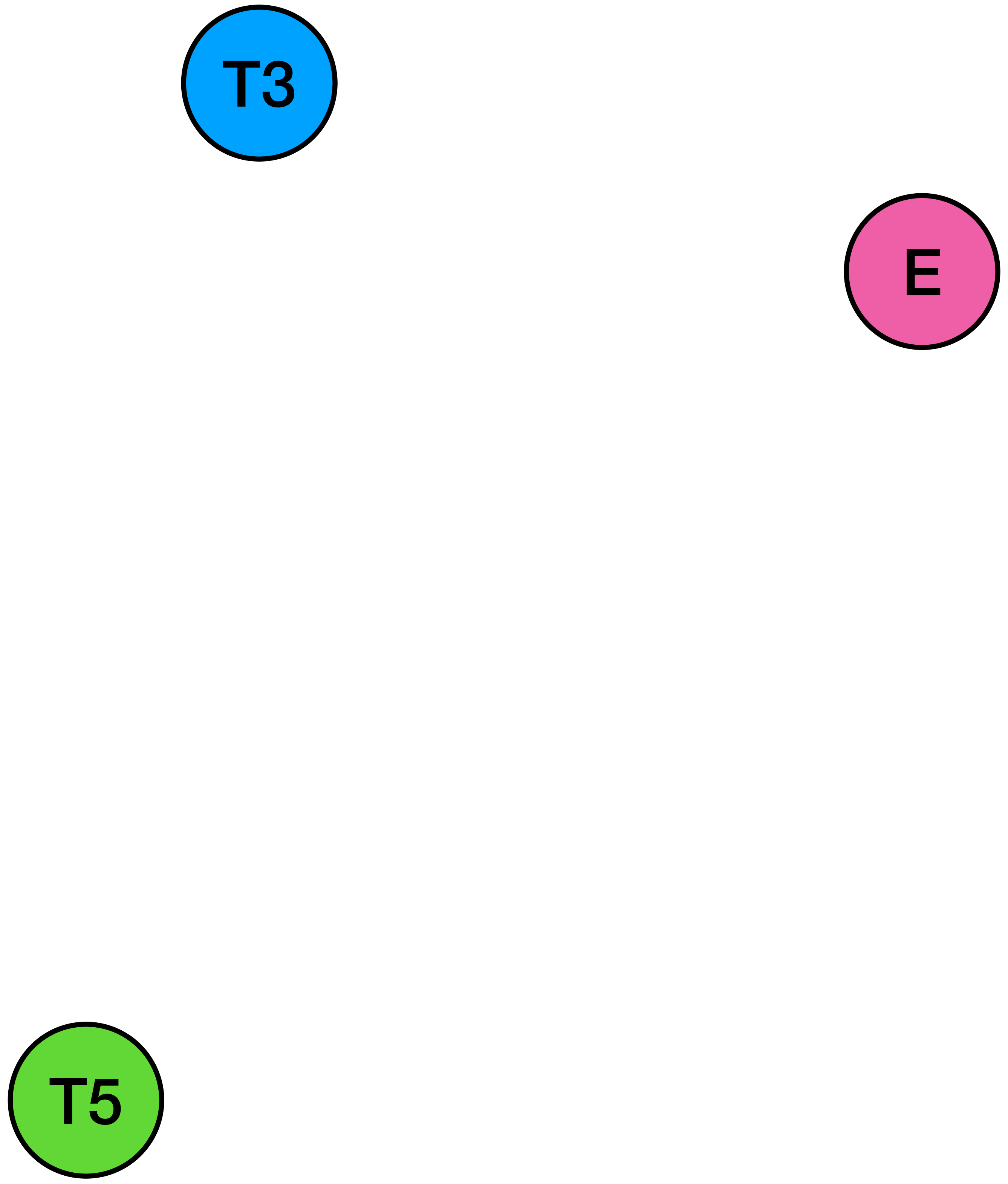

A
B
T1
T2
D
C
T4

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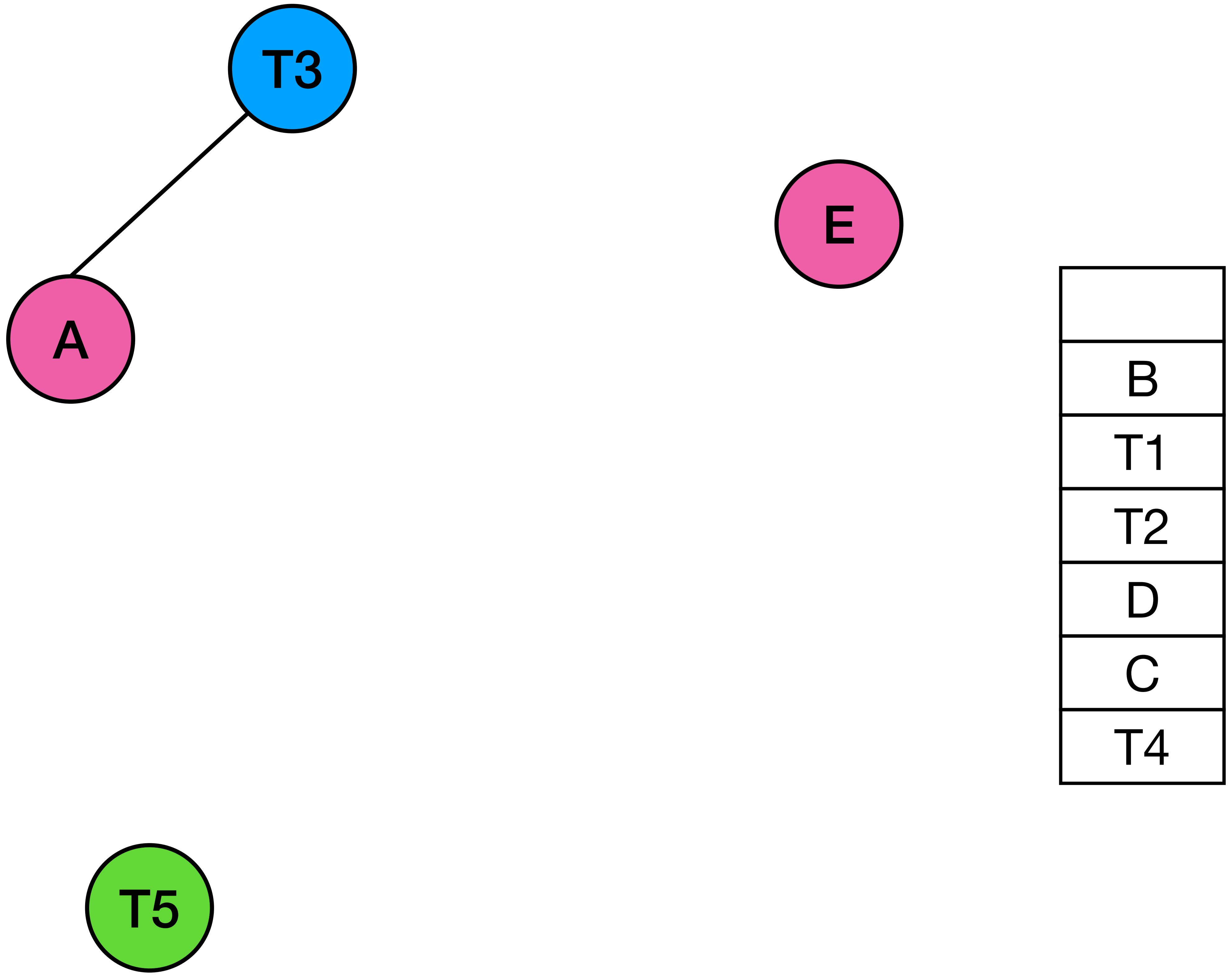

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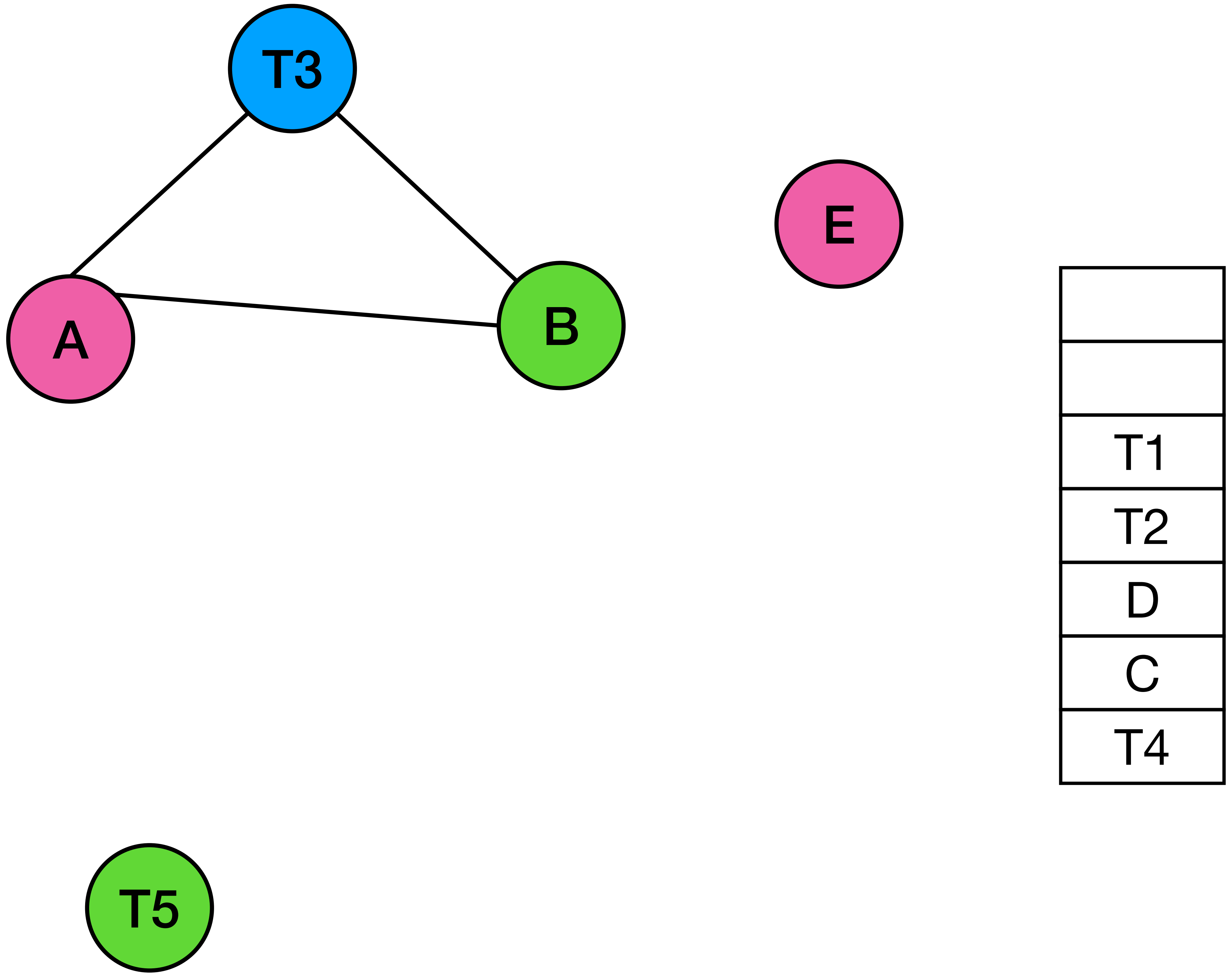

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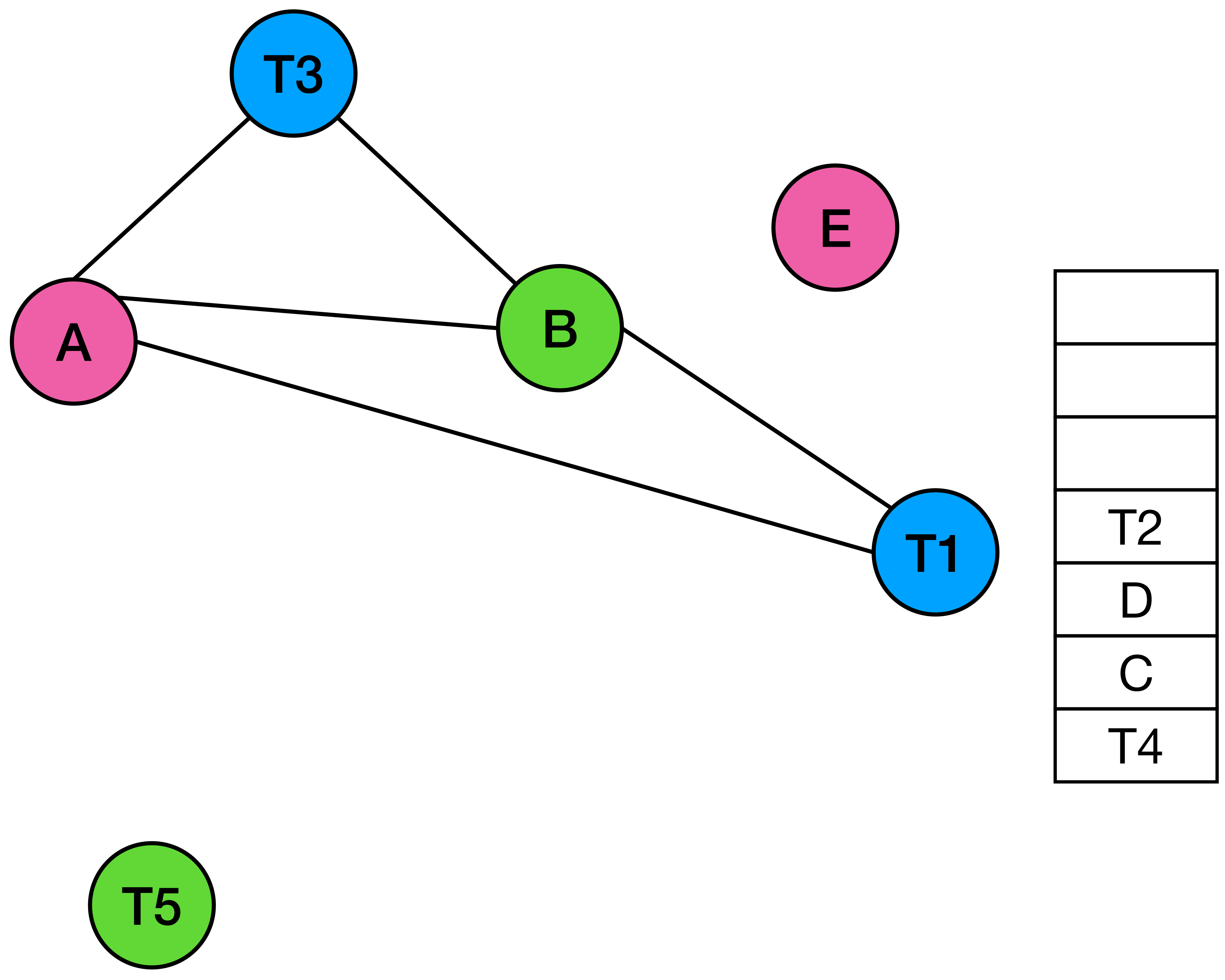

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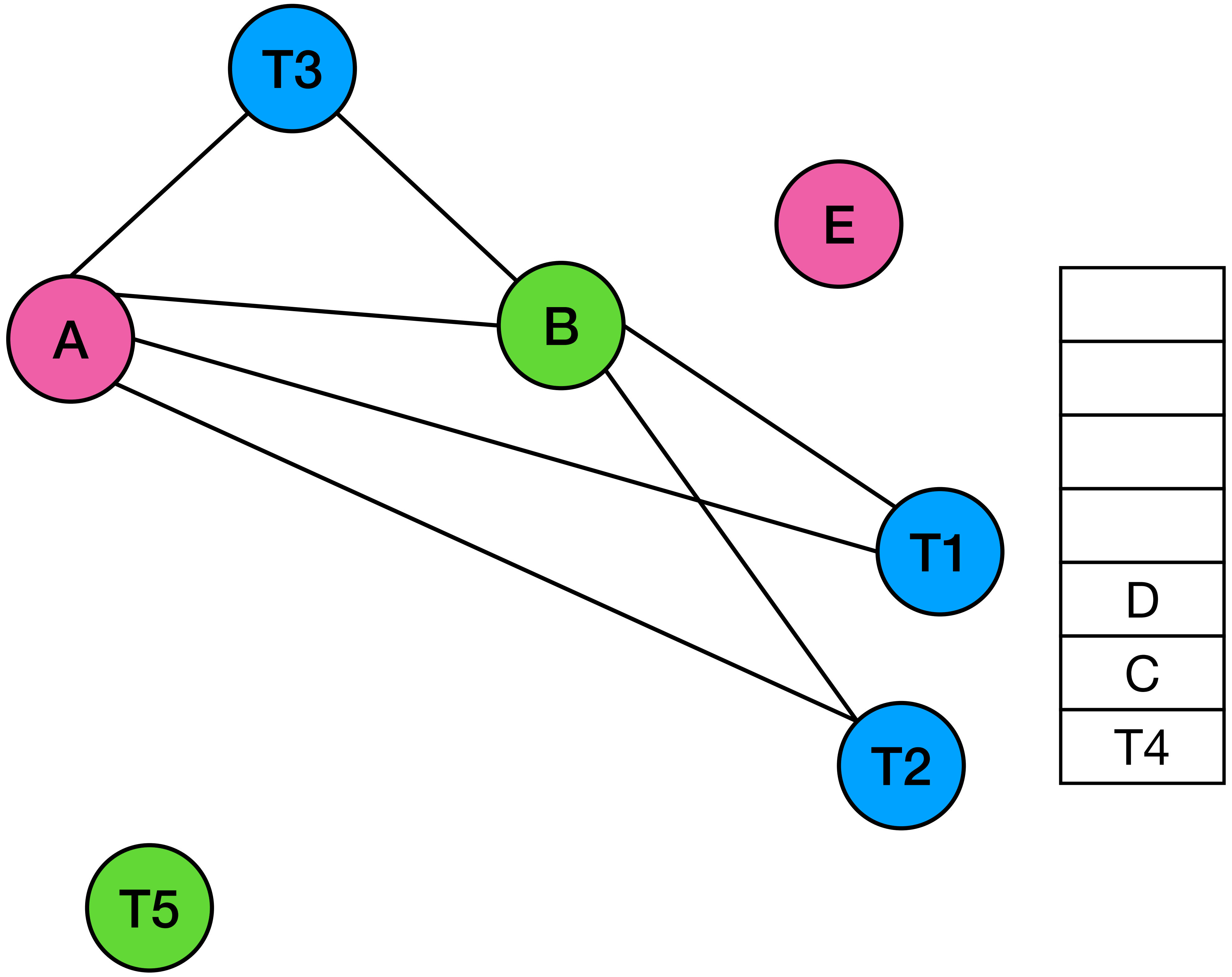

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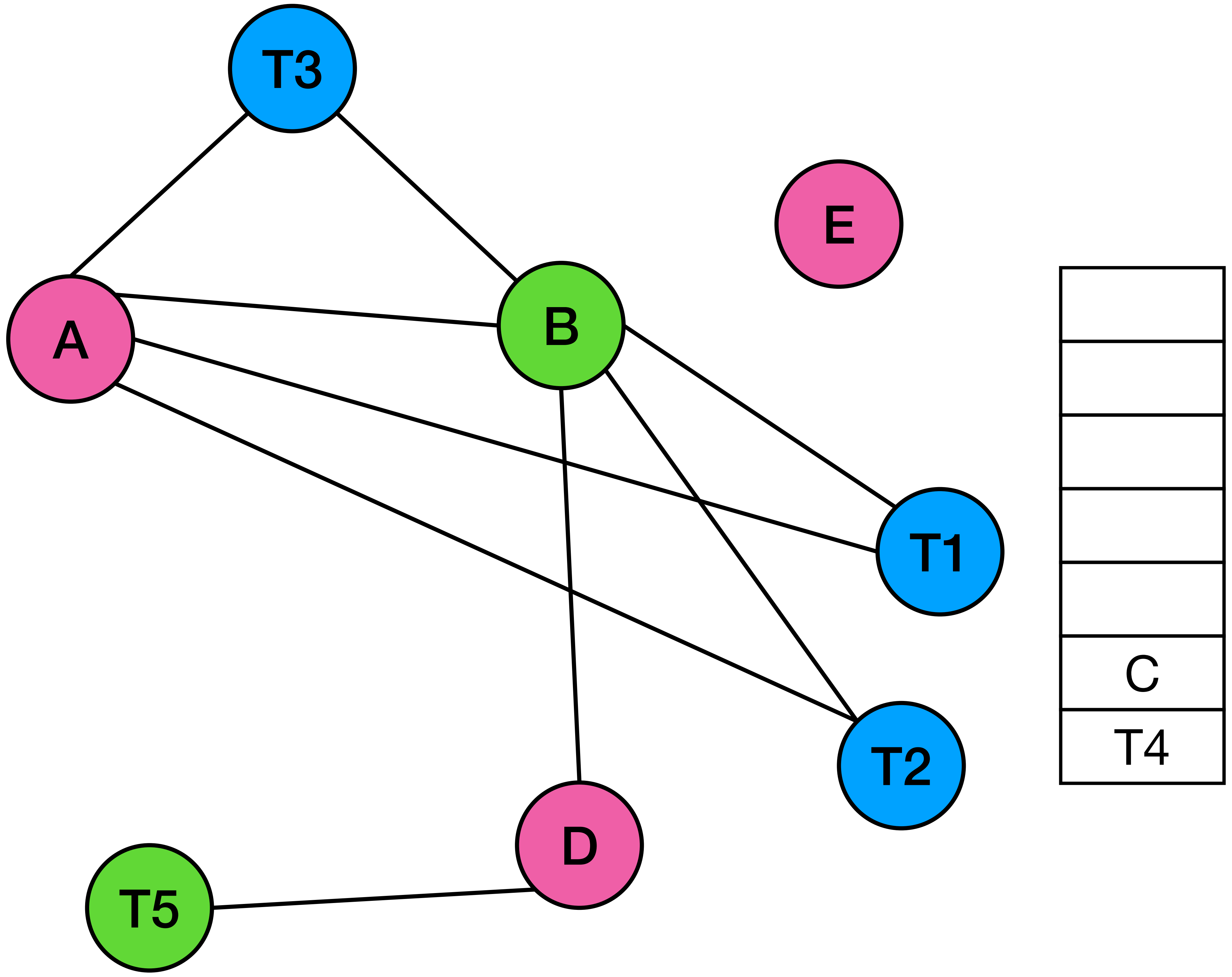

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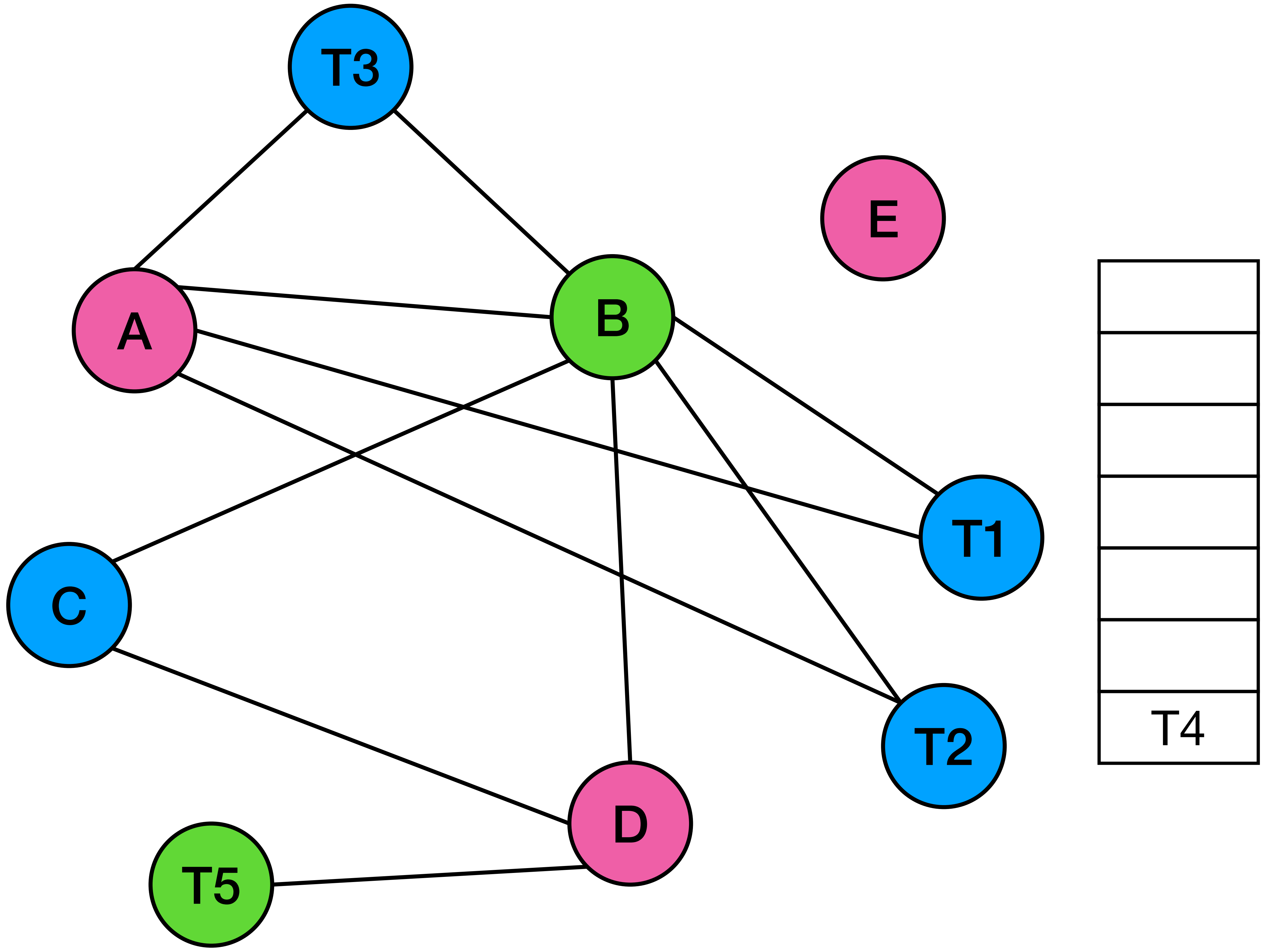

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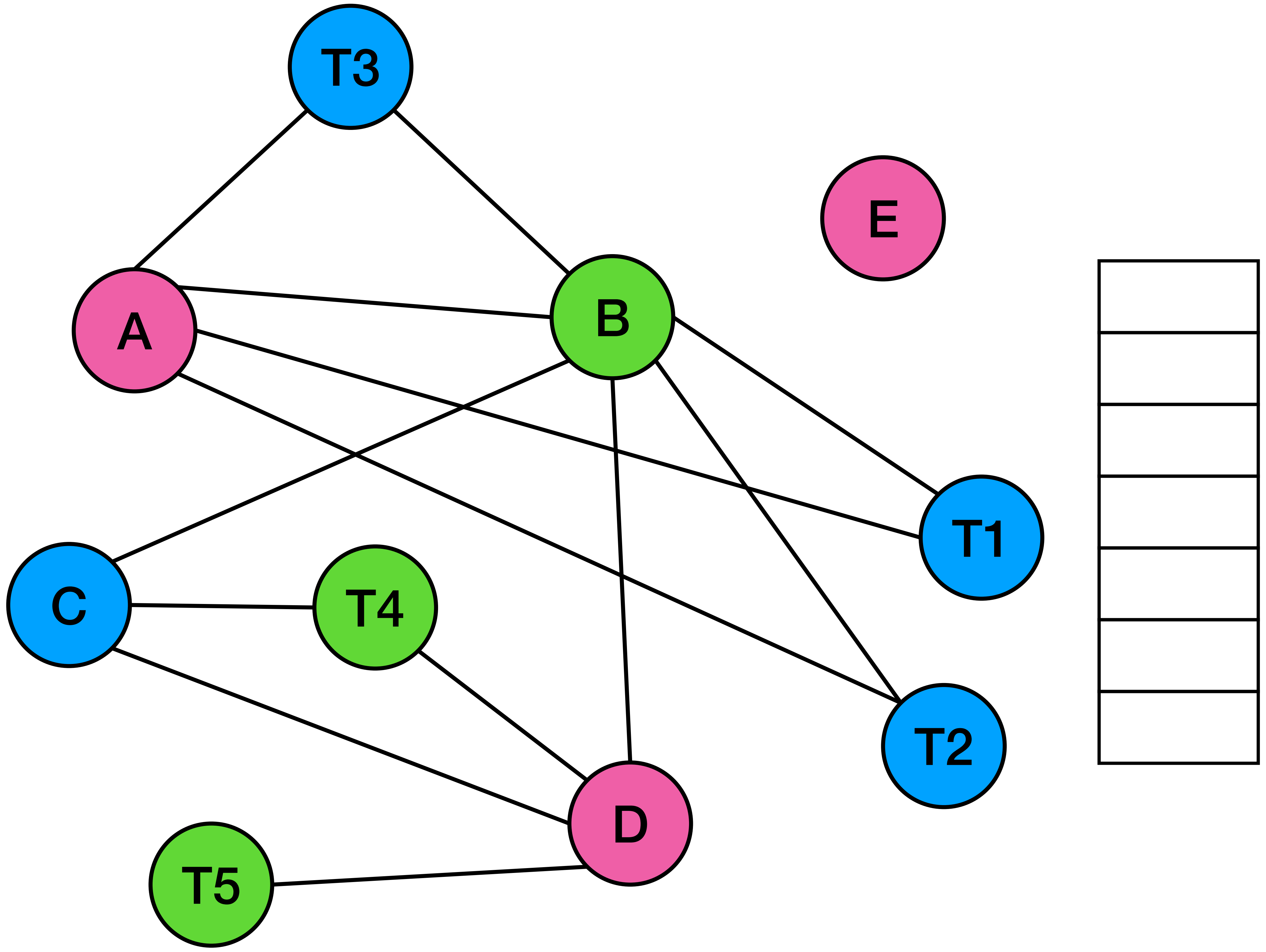
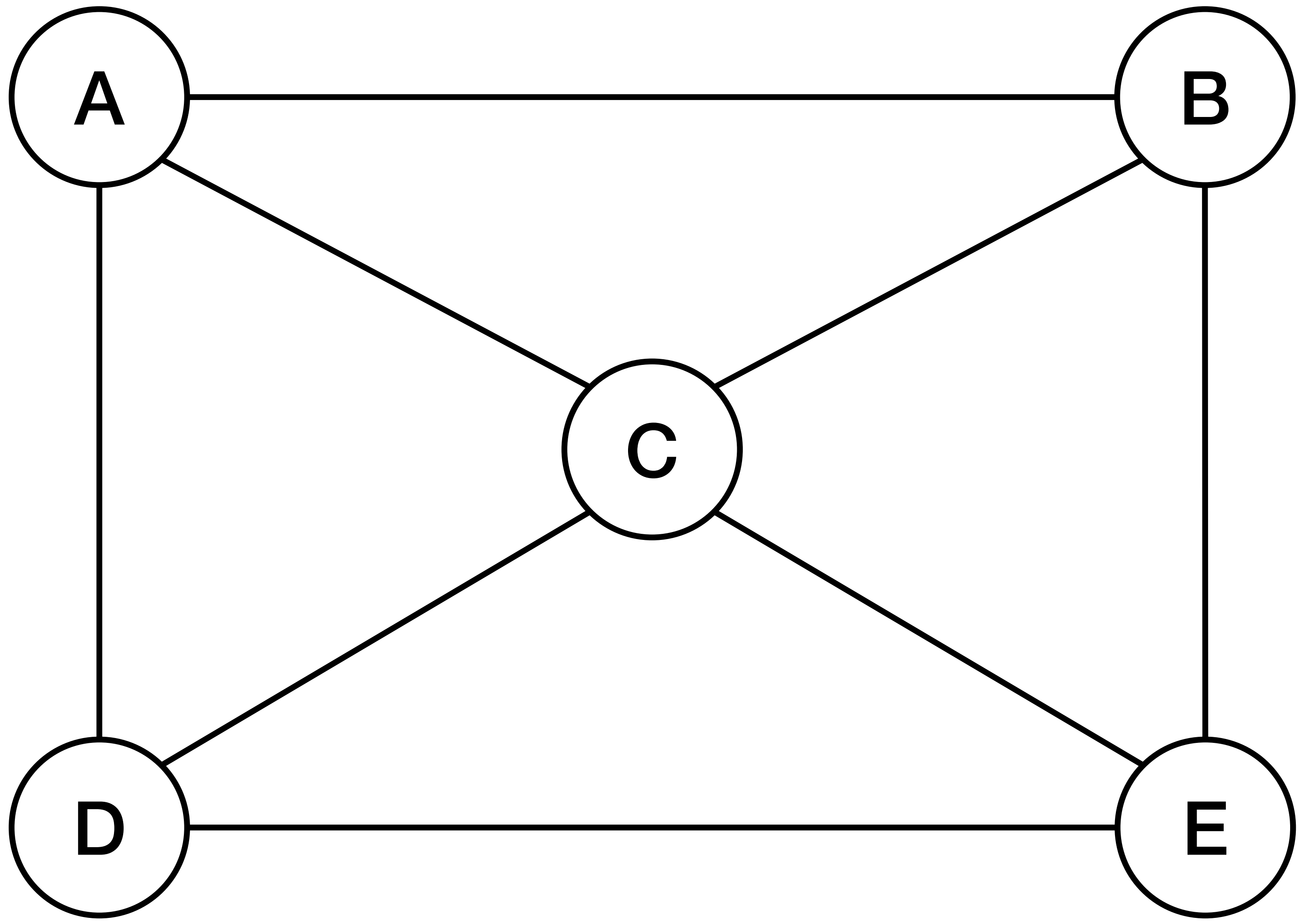


does this always work?

- What if there isn't a node to remove in step 1?
- Doesn't mean the graph can't be colored



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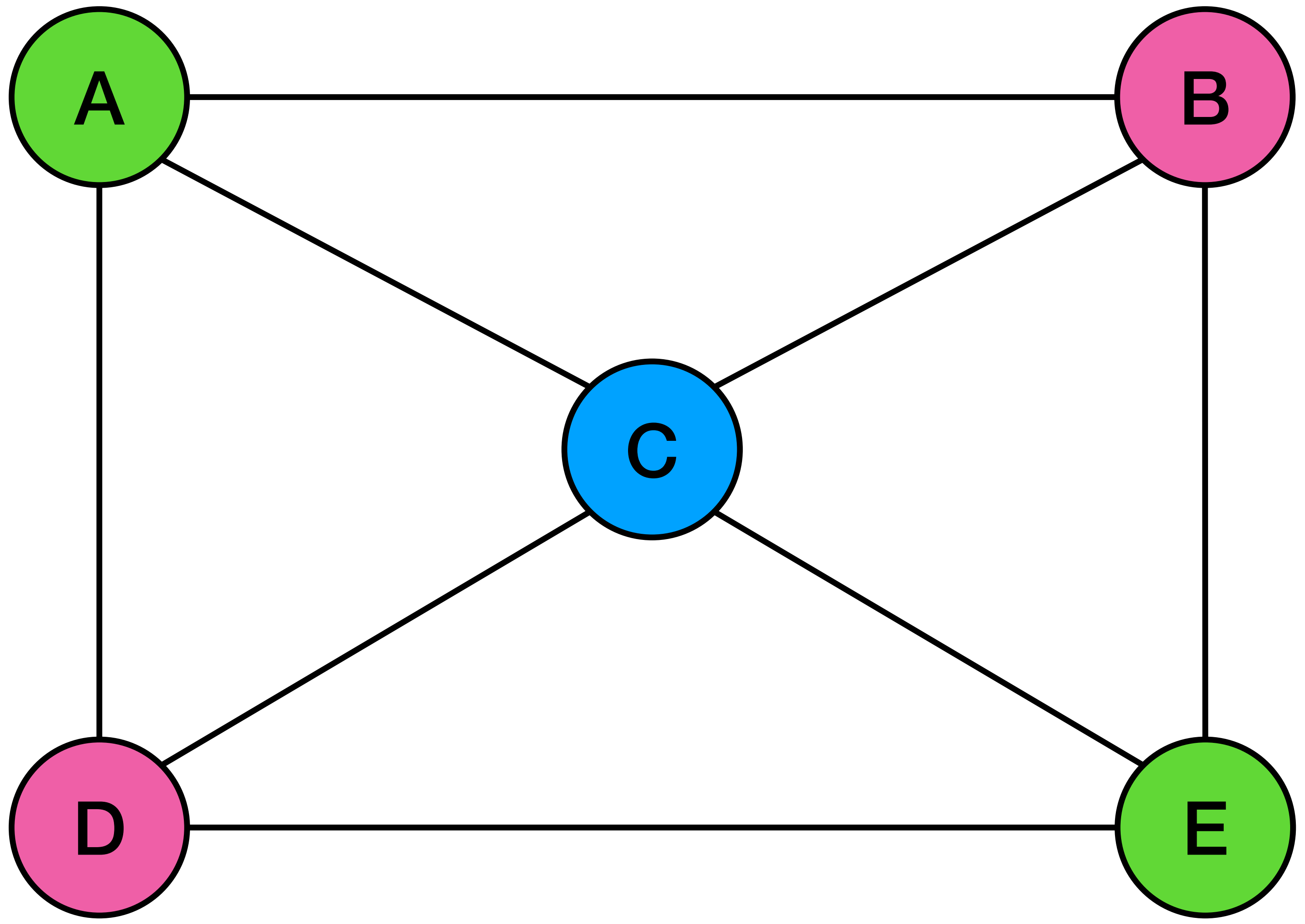
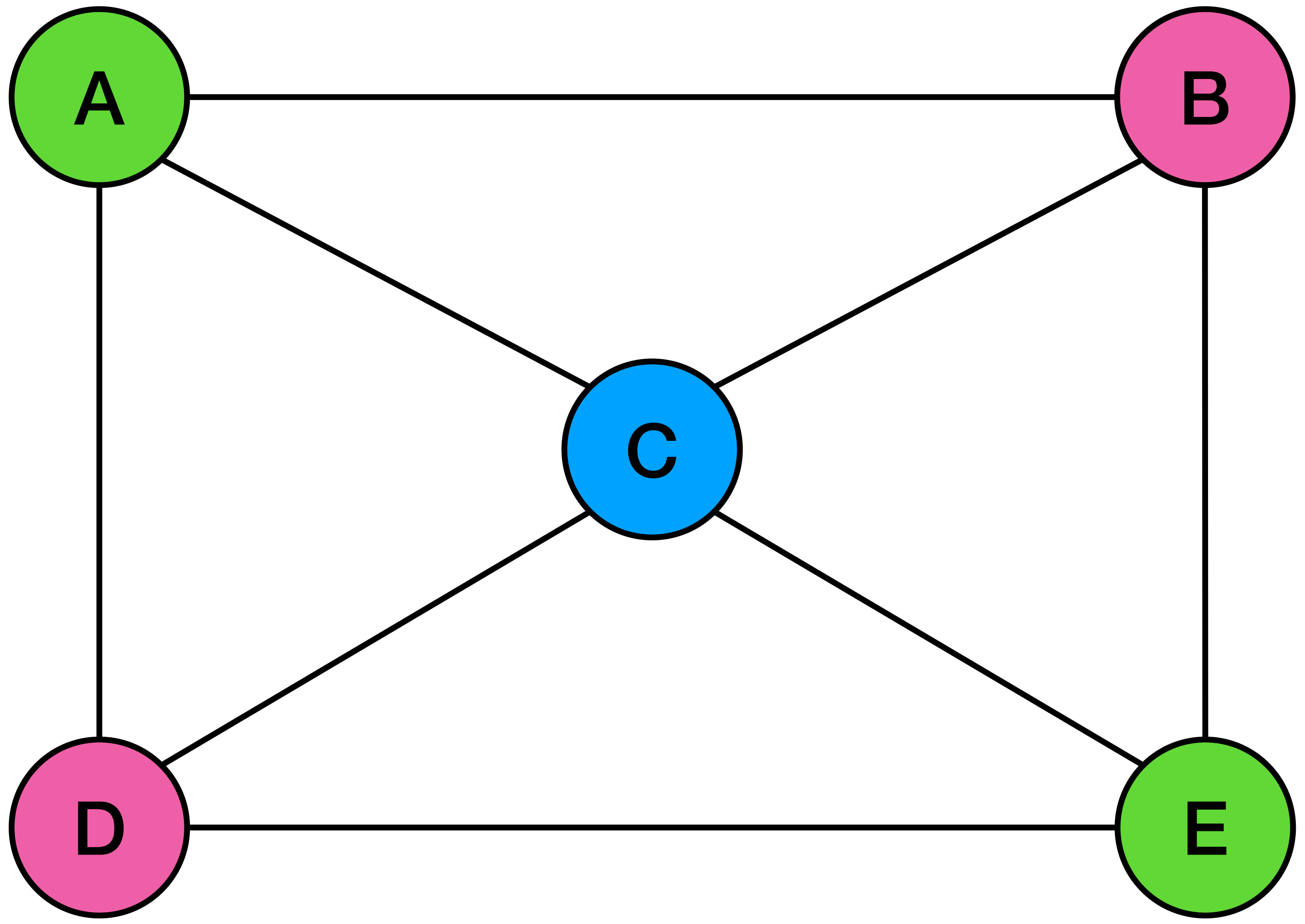


does this always work?

- Modified algorithm:
  - If no node can be safely removed, pick one anyway, mark it as a **potential spill**
  - Keep going
- If graph *still* can't be colored, need to deal with potential spill



next: dealing with spills