Graph Coloring Theory
graph coloring theory

• How do we color graphs?

1: T1 = A + B
2: T2 = A + T1
3: T3 = A + T2
4: D = A + T3
5: T4 = C + B
6: T5 = T4 + C
7: E = T5 + D
graph coloring theory

- Can we find a coloring of a graph whenever possible?
- Can we efficiently find the optimum coloring of a graph?

Problem: optimal graph coloring is **NP-hard**
- (Decision problem: can a graph be colored with K or fewer colors?)
kempe’s algorithm

• Algorithm from 1879 for finding a K-coloring of a graph

• Step 1: **Simplify**
  • Find a node with at most K-1 edges and remove it from the graph
  • Remember this node on the stack

• Observation: if smaller graph can be colored, bigger graph can be colored too (why?)
kempe’s algorithm

- Algorithm from 1879 for finding a K-coloring of a graph
- Step 1: **Simplify**
  - Find a node with at most K-1 edges and remove it from the graph
  - Remember this node on the stack
- Observation: if smaller graph can be colored, bigger graph can be colored too (why?)
kempe’s algorithm

- Algorithm from 1879 for finding a K-coloring of a graph
- Step 2: **Color**
  - Once smaller graph has been colored, add node back in
  - Assign a color
kempe’s algorithm

• Algorithm from 1879 for finding a K-coloring of a graph
• Step 2: **Color**
  • Once smaller graph has been colored, add node back in
  • Assign a color
Kempe’s algorithm

- Algorithm from 1879 for finding a K-coloring of a graph
- Apply steps 1 and 2 recursively:
  - Reduce graph
  - Color reduced graph if fewer than K vertices
  - Add nodes back into graph in reverse order they were removed
Kempe’s algorithm

- Algorithm from 1879 for finding a $K$-coloring of a graph
- Apply steps 1 and 2 recursively:
  - Reduce graph
  - Color reduced graph if fewer than $K$ vertices
  - Add nodes back into graph in reverse order they were removed
kempe’s algorithm

• Algorithm from 1879 for finding a K-coloring of a graph
• Apply steps 1 and 2 recursively:
  • Reduce graph
  • Color reduced graph if fewer than K vertices
  • Add nodes back into graph in reverse order they were removed
kempe’s algorithm

• Algorithm from 1879 for finding a K-coloring of a graph
• Apply steps 1 and 2 recursively:
  • Reduce graph
  • Color reduced graph if fewer than K vertices
  • Add nodes back into graph in reverse order they were removed
kempe’s algorithm

- Algorithm from 1879 for finding a K-coloring of a graph
- Apply steps 1 and 2 recursively:
  - Reduce graph
  - Color reduced graph if fewer than K vertices
  - Add nodes back into graph in reverse order they were removed
kempe’s algorithm

- Algorithm from 1879 for finding a K-coloring of a graph
- Apply steps 1 and 2 recursively:
  - Reduce graph
  - Color reduced graph if fewer than K vertices
  - Add nodes back into graph in reverse order they were removed
kempe’s algorithm

- Algorithm from 1879 for finding a K-coloring of a graph
- Apply steps 1 and 2 recursively:
  - Reduce graph
  - Color reduced graph if fewer than K vertices
  - Add nodes back into graph in reverse order they were removed
kempe’s algorithm

• Algorithm from 1879 for finding a K-coloring of a graph
• Apply steps 1 and 2 recursively:
  • Reduce graph
  • Color reduced graph if fewer than K vertices
  • Add nodes back into graph in reverse order they were removed
kempe’s algorithm

• Algorithm from 1879 for finding a K-coloring of a graph
• Apply steps 1 and 2 recursively:
  • Reduce graph
  • Color reduced graph if fewer than K vertices
  • Add nodes back into graph in reverse order they were removed
kempe’s algorithm

- Algorithm from 1879 for finding a K-coloring of a graph
- Apply steps 1 and 2 recursively:
  - Reduce graph
  - Color reduced graph if fewer than K vertices
  - Add nodes back into graph in reverse order they were removed
kempe’s algorithm

- Algorithm from 1879 for finding a K-coloring of a graph
- Apply steps 1 and 2 recursively:
  - Reduce graph
  - Color reduced graph if fewer than K vertices
  - Add nodes back into graph in reverse order they were removed
kempe’s algorithm

• Algorithm from 1879 for finding a K-coloring of a graph
• Apply steps 1 and 2 recursively:
  • Reduce graph
  • Color reduced graph if fewer than K vertices
  • Add nodes back into graph in reverse order they were removed
kempe’s algorithm

- Algorithm from 1879 for finding a K-coloring of a graph
- Apply steps 1 and 2 recursively:
  - Reduce graph
  - Color reduced graph if fewer than K vertices
  - Add nodes back into graph in reverse order they were removed
Kempe’s Algorithm

- Algorithm from 1879 for finding a K-coloring of a graph
- Apply steps 1 and 2 recursively:
  - Reduce graph
  - Color reduced graph if fewer than K vertices
  - Add nodes back into graph in reverse order they were removed
kempe’s algorithm

- Algorithm from 1879 for finding a K-coloring of a graph
- Apply steps 1 and 2 recursively:
  - Reduce graph
  - Color reduced graph if fewer than K vertices
  - Add nodes back into graph in reverse order they were removed
kempe’s algorithm

• Algorithm from 1879 for finding a K-coloring of a graph
• Apply steps 1 and 2 recursively:
  • Reduce graph
  • Color reduced graph if fewer than K vertices
  • Add nodes back into graph in reverse order they were removed
Kempe’s algorithm

- Algorithm from 1879 for finding a K-coloring of a graph
- Apply steps 1 and 2 recursively:
  - Reduce graph
  - Color reduced graph if fewer than K vertices
  - Add nodes back into graph in reverse order they were removed
does this always work?

- What if there isn’t a node to remove in step 1?
- Doesn’t mean the graph can’t be colored
does this always work?

- What if there isn’t a node to remove in step 1?
- Doesn’t mean the graph can’t be colored
does this always work?

• Modified algorithm:
  • If no node can be safely removed, pick one anyway, mark it as a potential spill
  • Keep going

• If graph still can’t be colored, need to deal with potential spill
next: dealing with spills