First and Follow Sets

first and follow sets

- Figuring out which token to look for to match a given rule is complicated
- But we can simplify this by computing **first** and **follow** sets
  - **First(α)** = what terminals (or λ) might *start* any string you derive from α
    - If I start with α and apply rules, what terminals might the string start with?
  - **Follow(X)** = what terminals might *come after* the non-terminal X
    - If I start with the *start symbol* and apply rules, what terminals can I make come after X?
- We are going to figure out how to build first and follow sets by solving **systems of set constraints**

set constraints

- Can compute first and follow sets by solving **set constraints**
  - $X \subseteq Y$: X is a subset of Y (Y contains everything in X, and maybe more)
- A **system** of set constraints is one or more constraints on sets:

  $\{a\} \subseteq X$ (X contains a; can also be written $a \in X$)

  $X \subseteq Y$ (Y is a superset of X)

  $b \in Z$ (Z contains b)

  $Z \subseteq Y$ (Z is a subset of Y)

- A solution to a system of constraints is an assignment of values to sets that satisfies each of the constraints:

  $$X = \{a\} \quad Y = \{a, b\} \quad Z = \{b\}$$

  (usually interested in the *least* solution — the solution using the smallest sets)

solving set constraints

- Iterative algorithm for solving set constraints: As we add new constraints, update sets to keep them consistent

- Create a graph with one vertex for each set

- $a \in X$: add a to the set X

- $X \subseteq Y$: add an edge from X to Y, any time we put something new in X, push it to Y

$\{a\} \subseteq X$
$X \subseteq Y$
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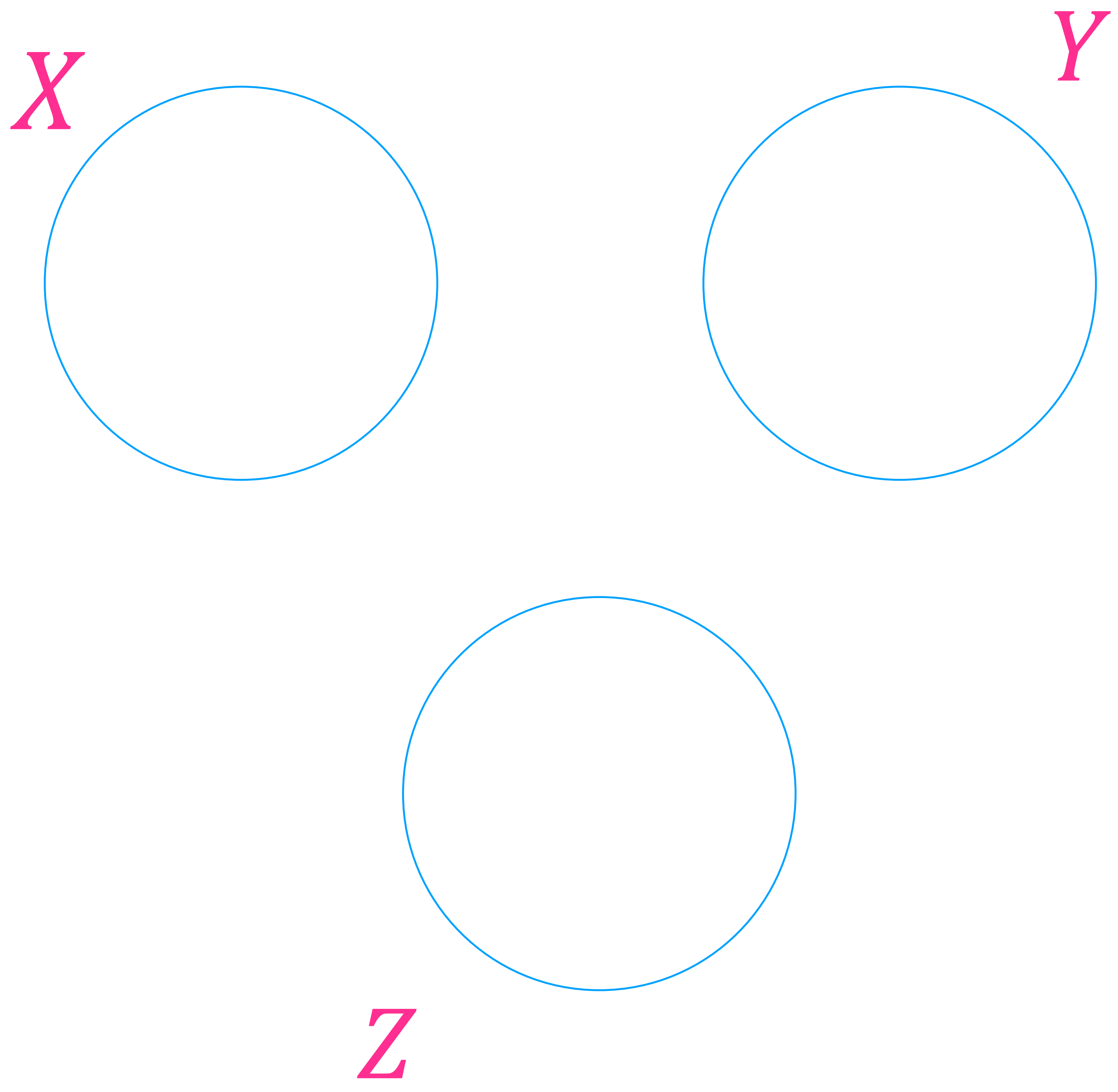

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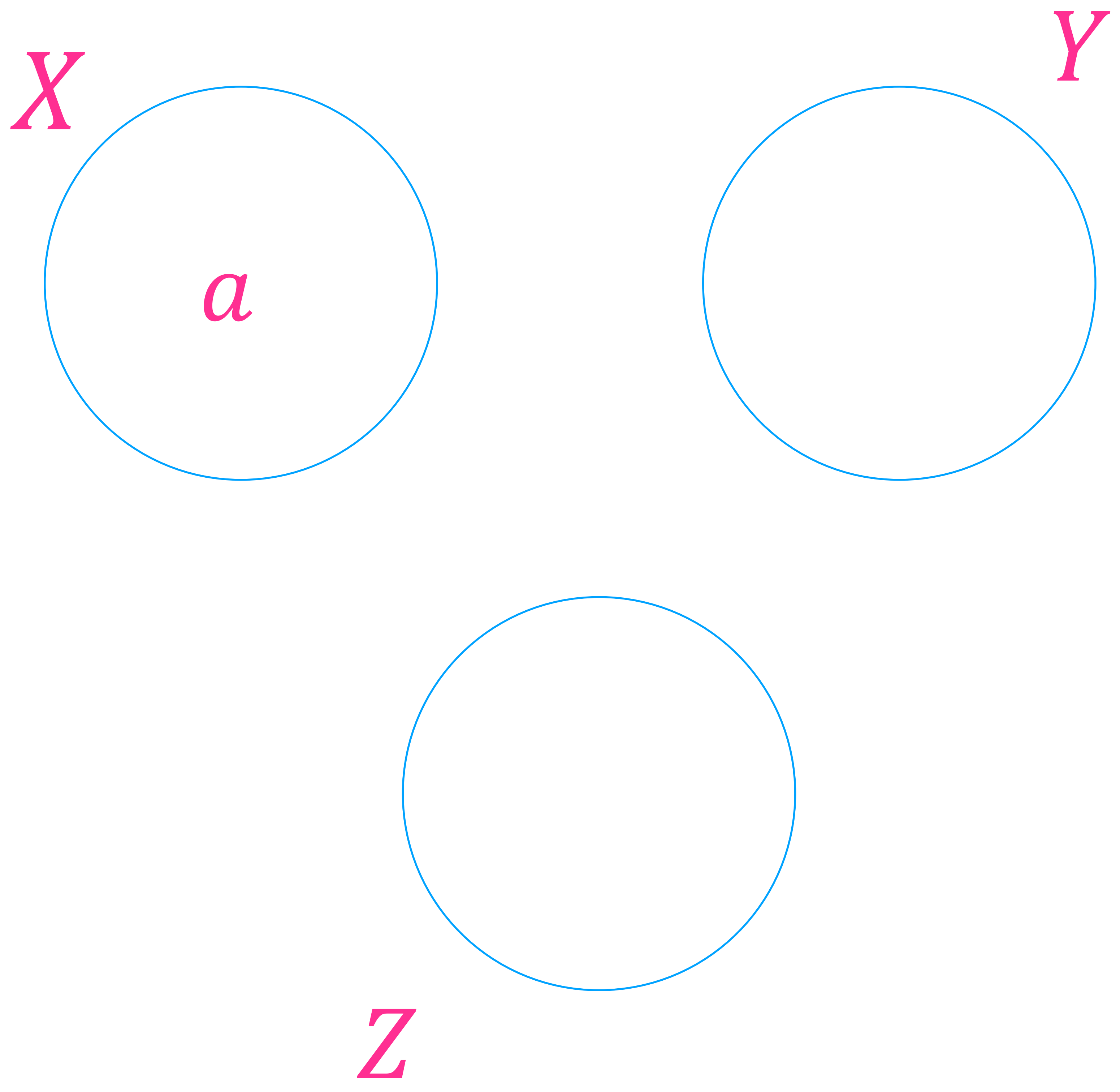

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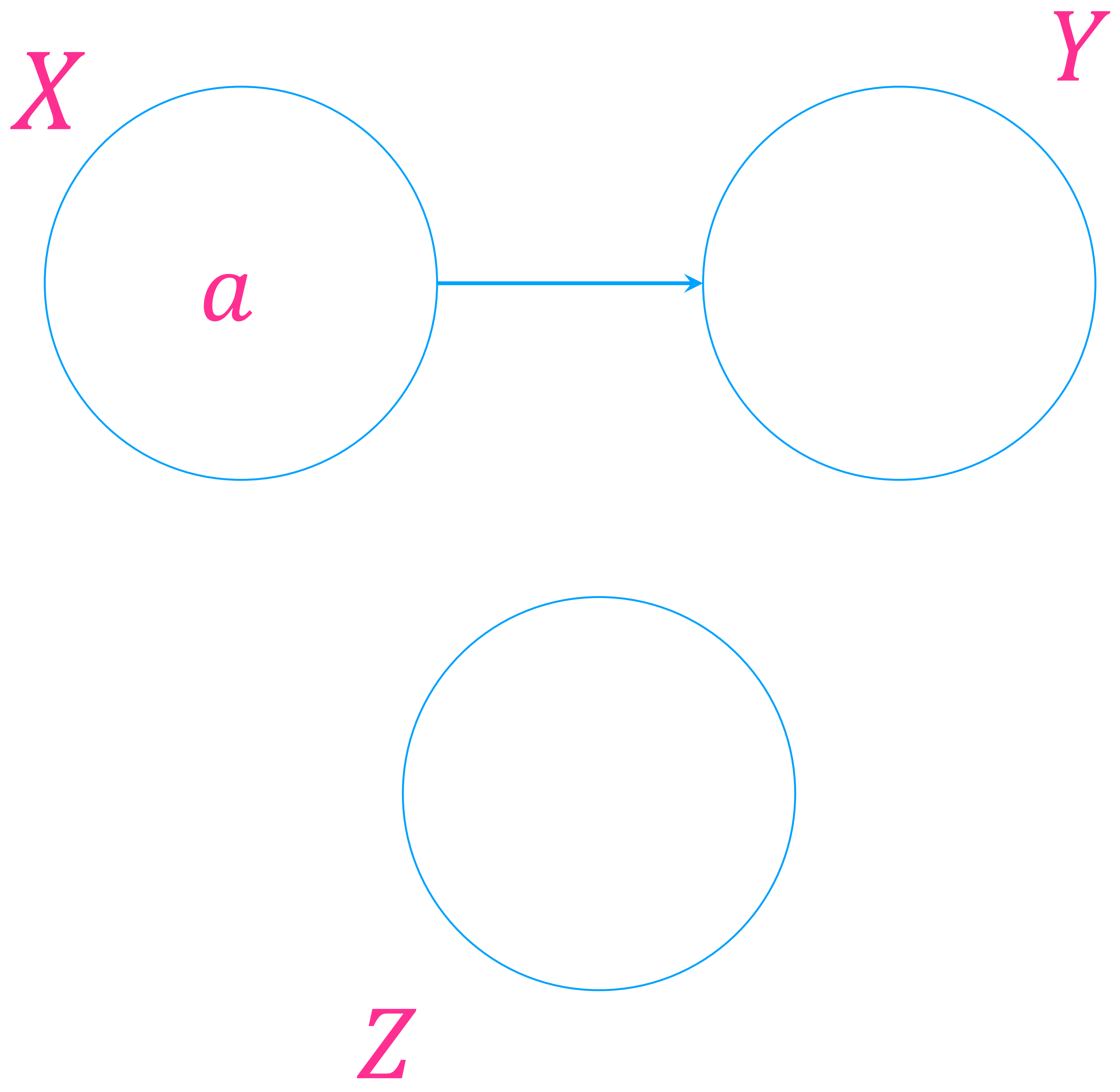

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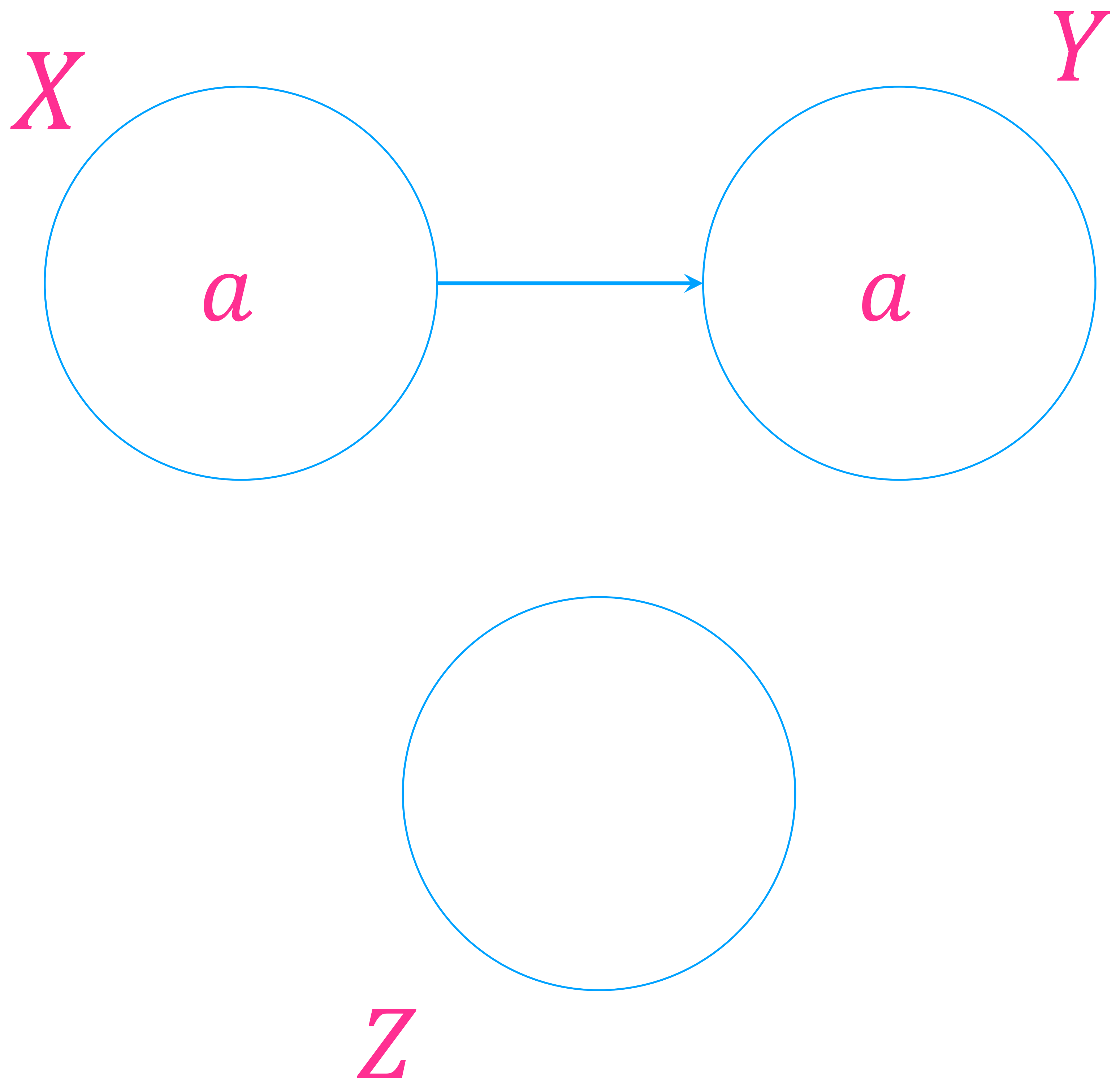

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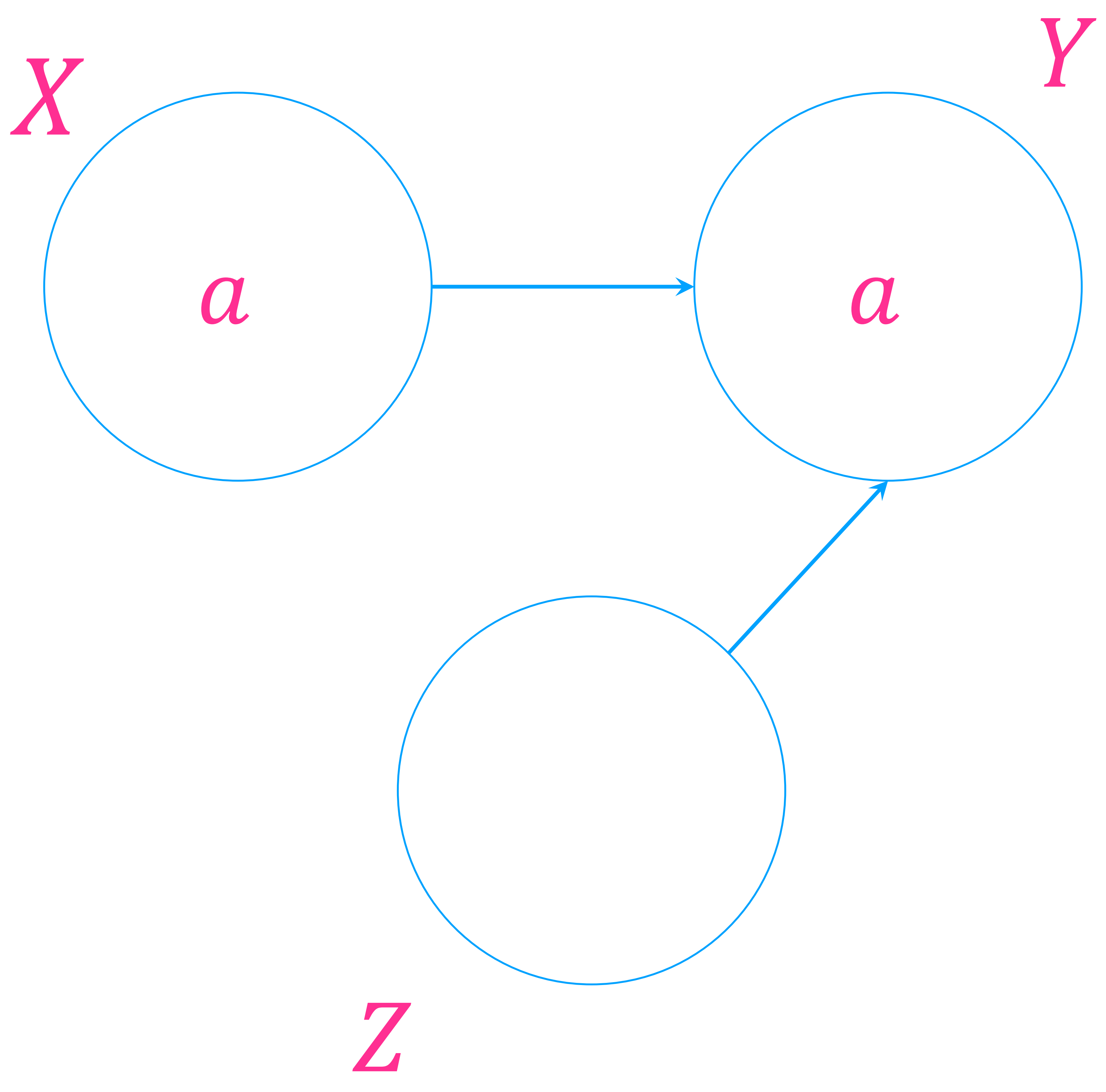

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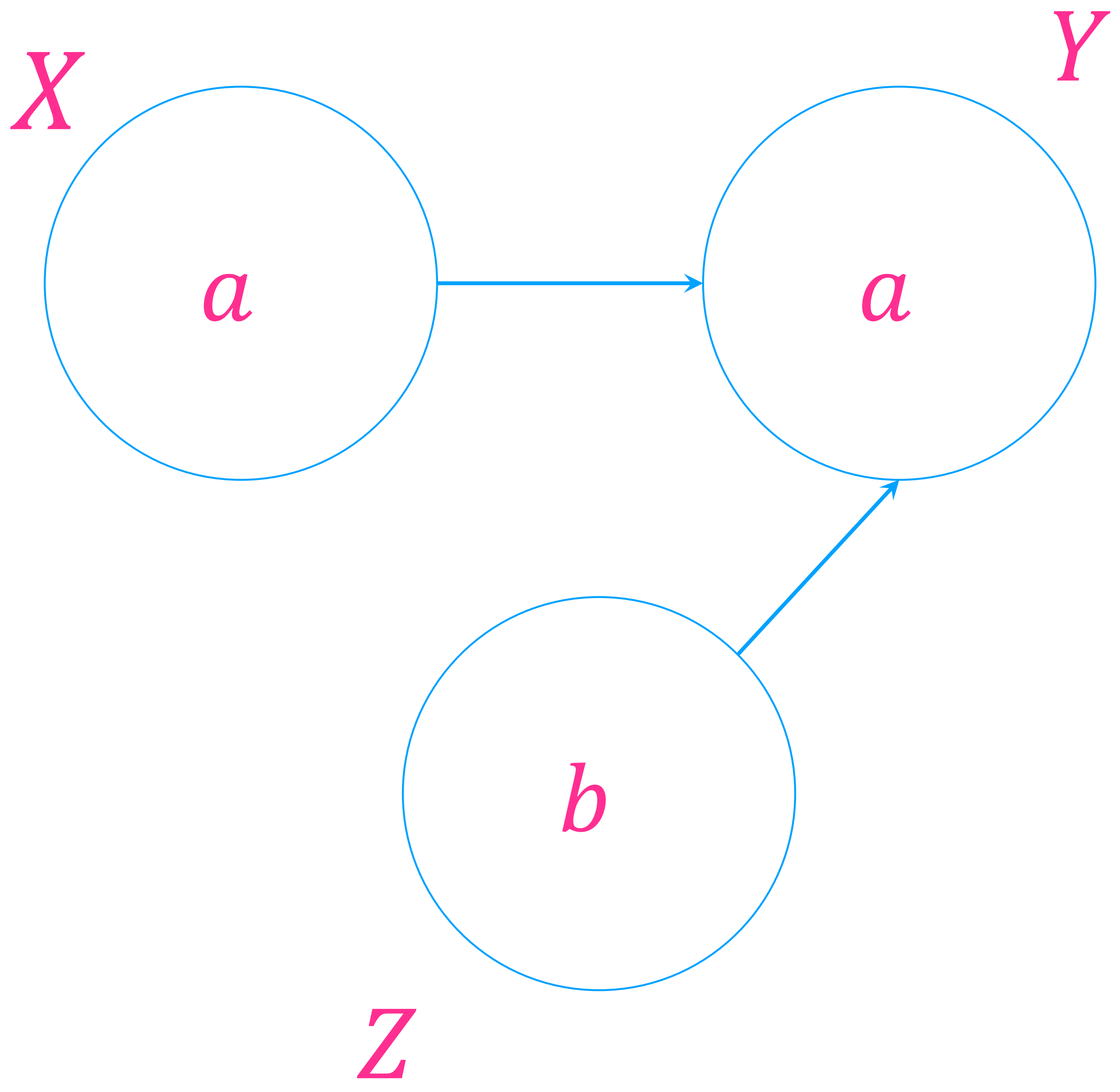

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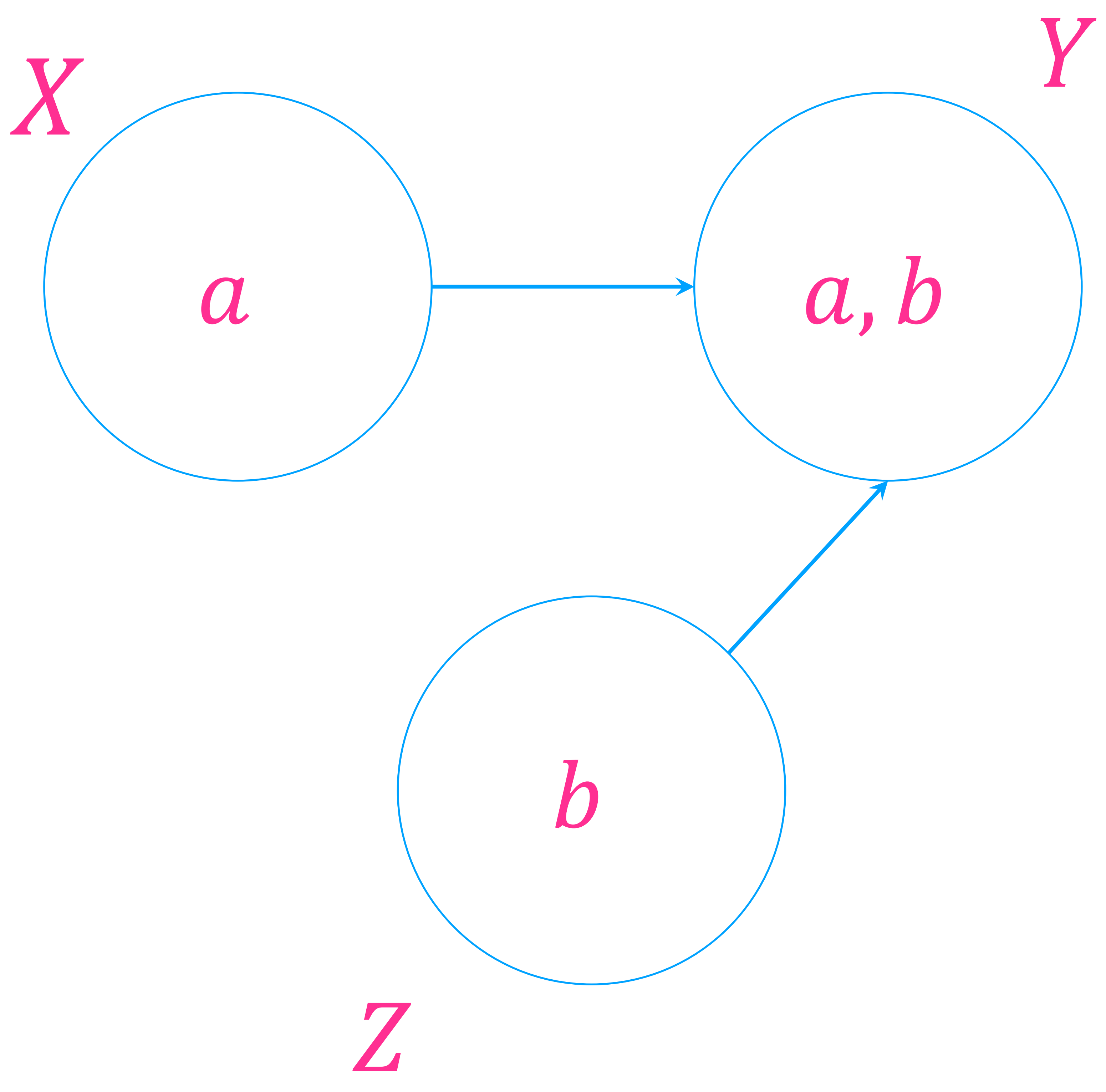

first sets

- First(a) if a is a terminal: {a}
- First(A) if A is a non-terminal:

Look at all productions for A
$A \to X_1 X_2 \ldots X_k$

1. First(A) ⊇ First(X_1) - λ
2. If λ ∈First(X_1) then First(A) ⊇ First(X_2) - λ and so on
3. If λ ∈First(X_i) for all i, then λ ∈First(A)

- Computing First sets for an arbitrary string works the same way

example first sets

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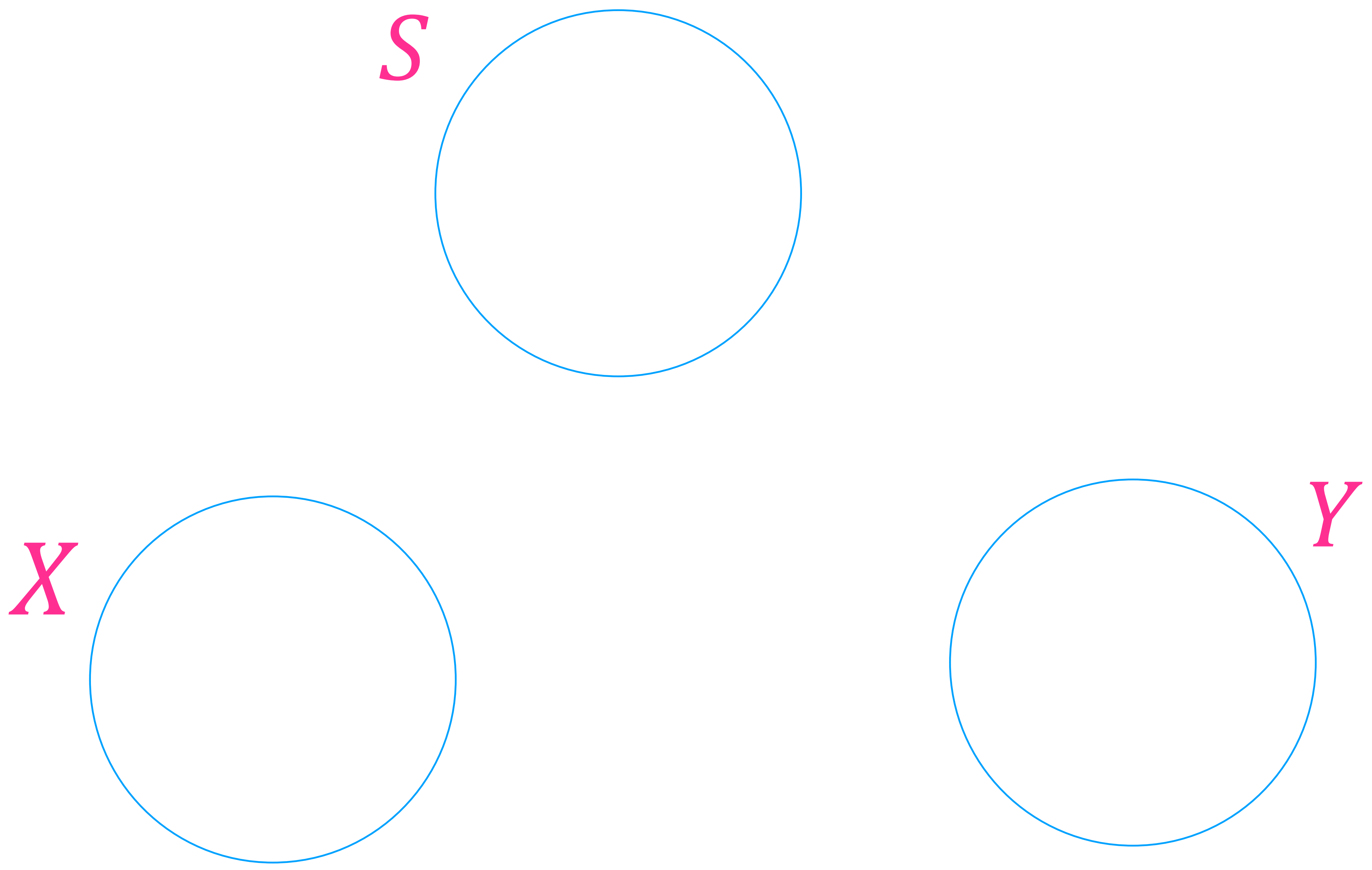

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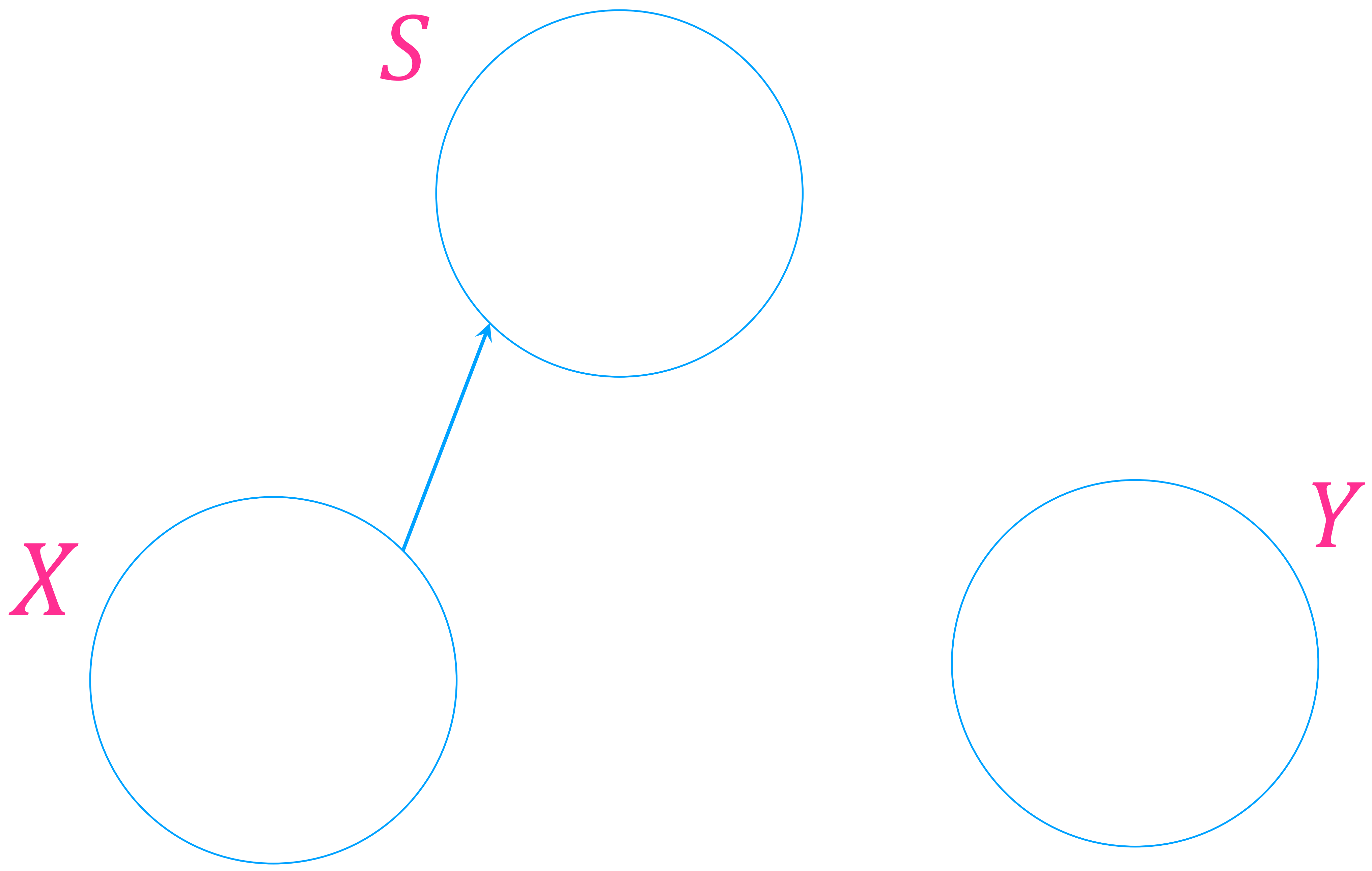

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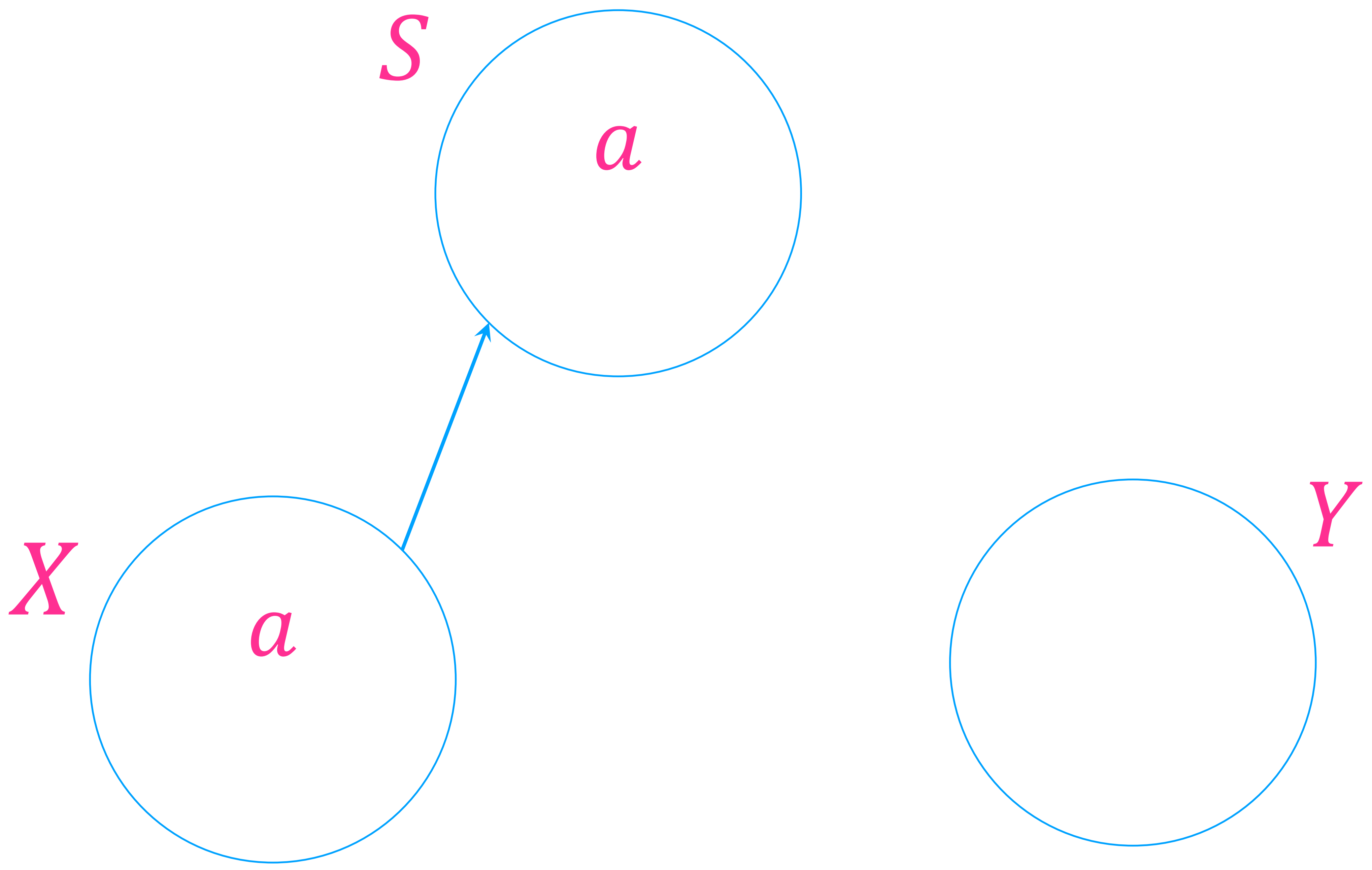

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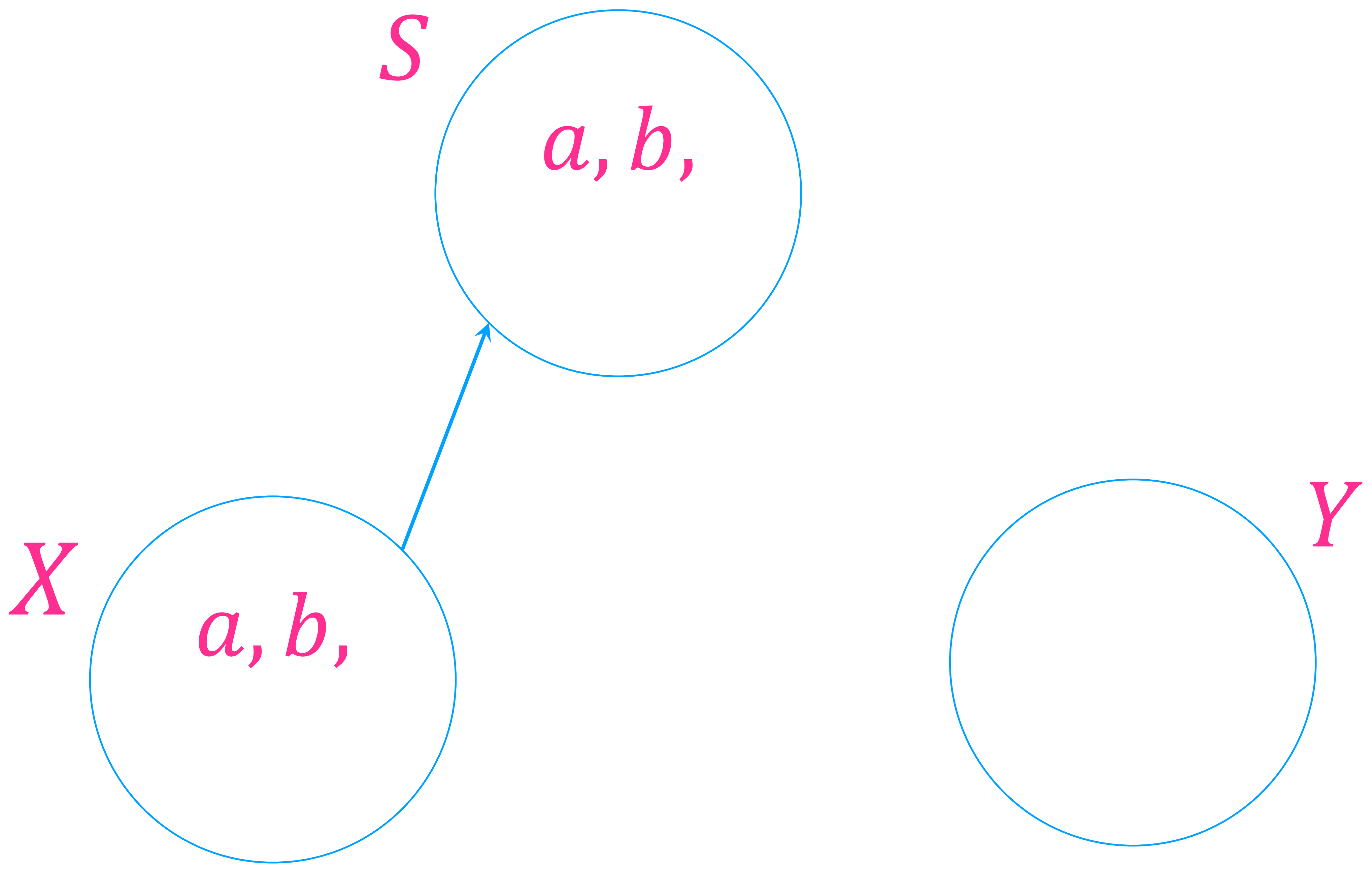

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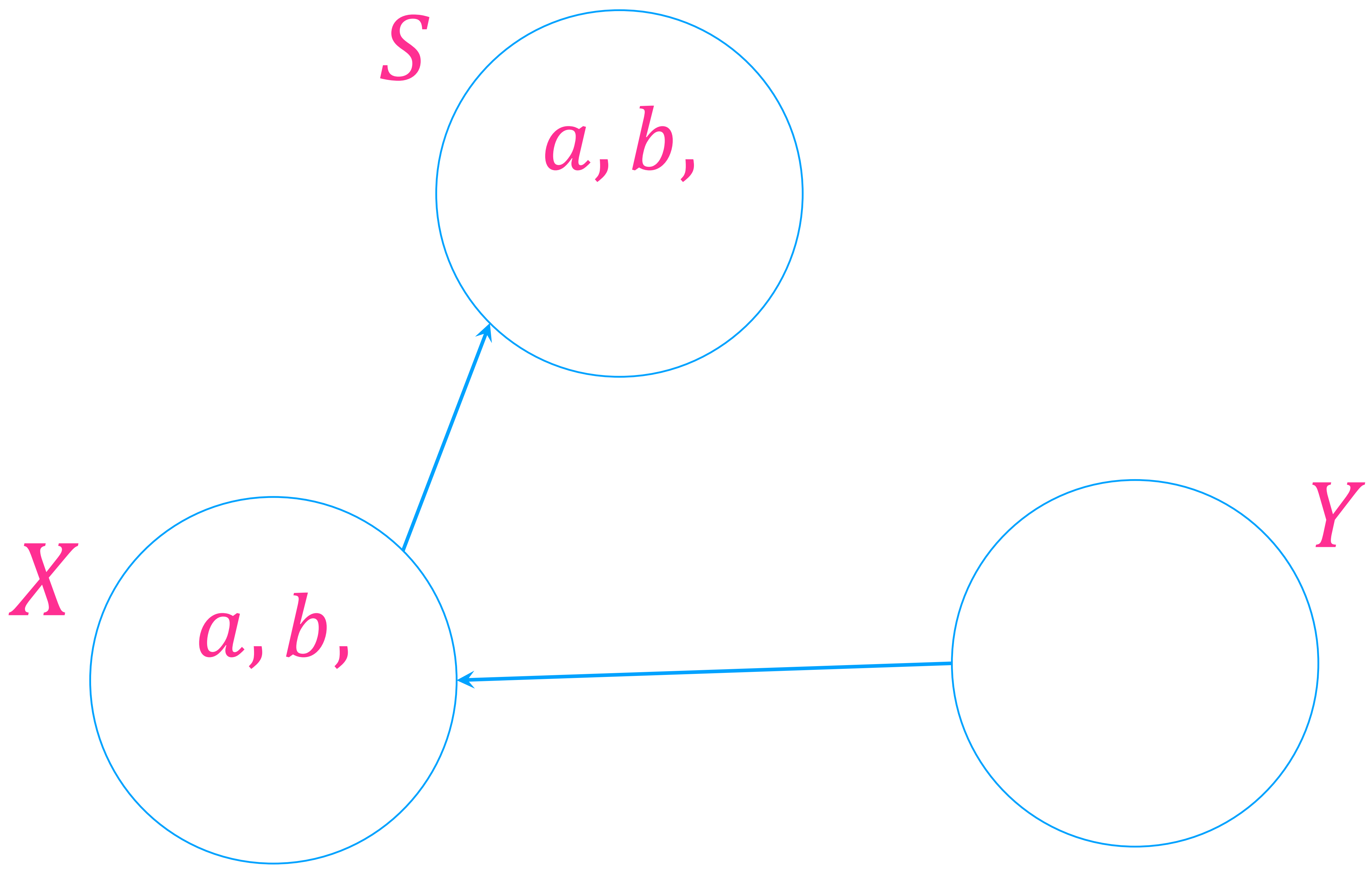

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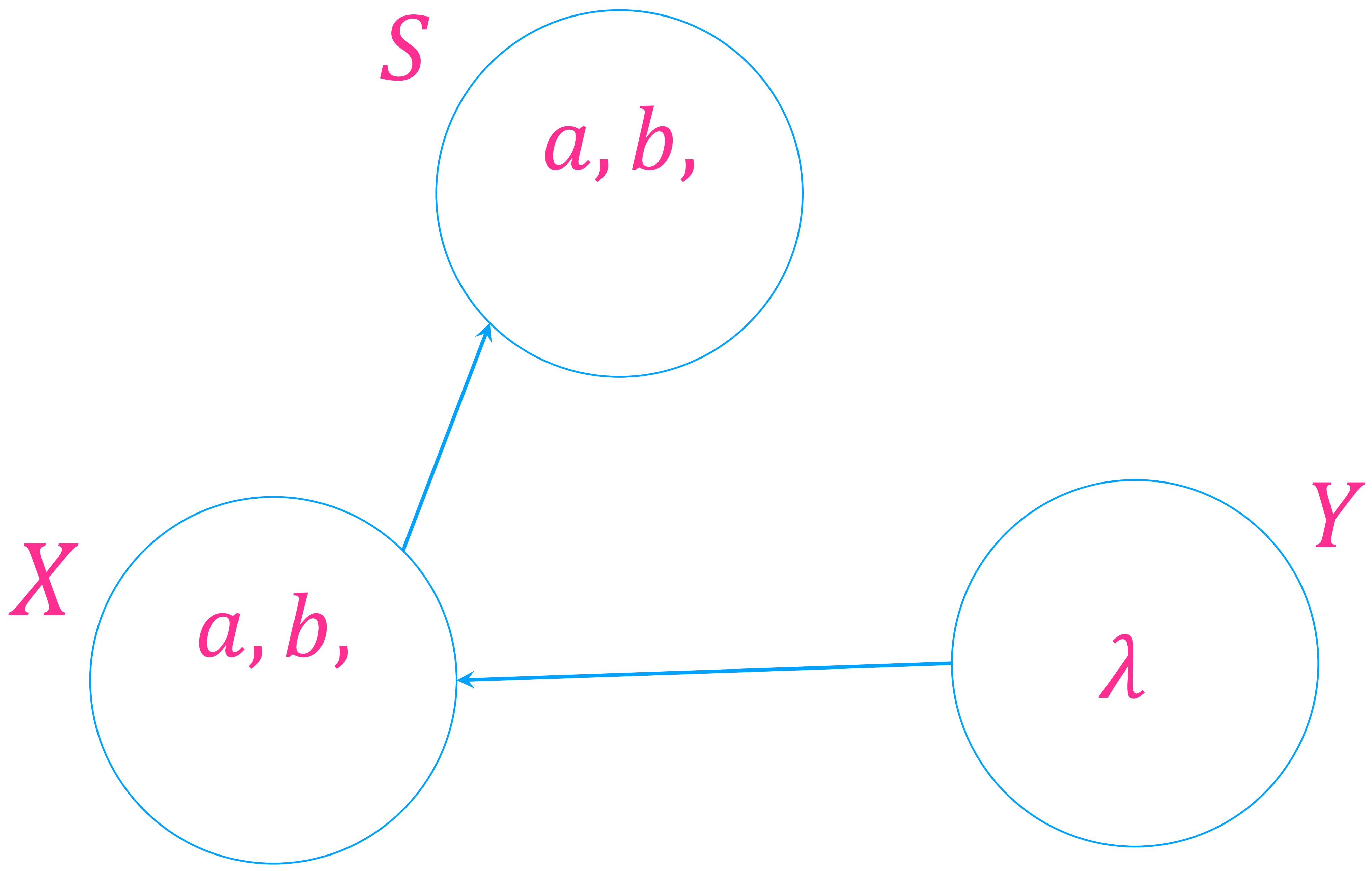

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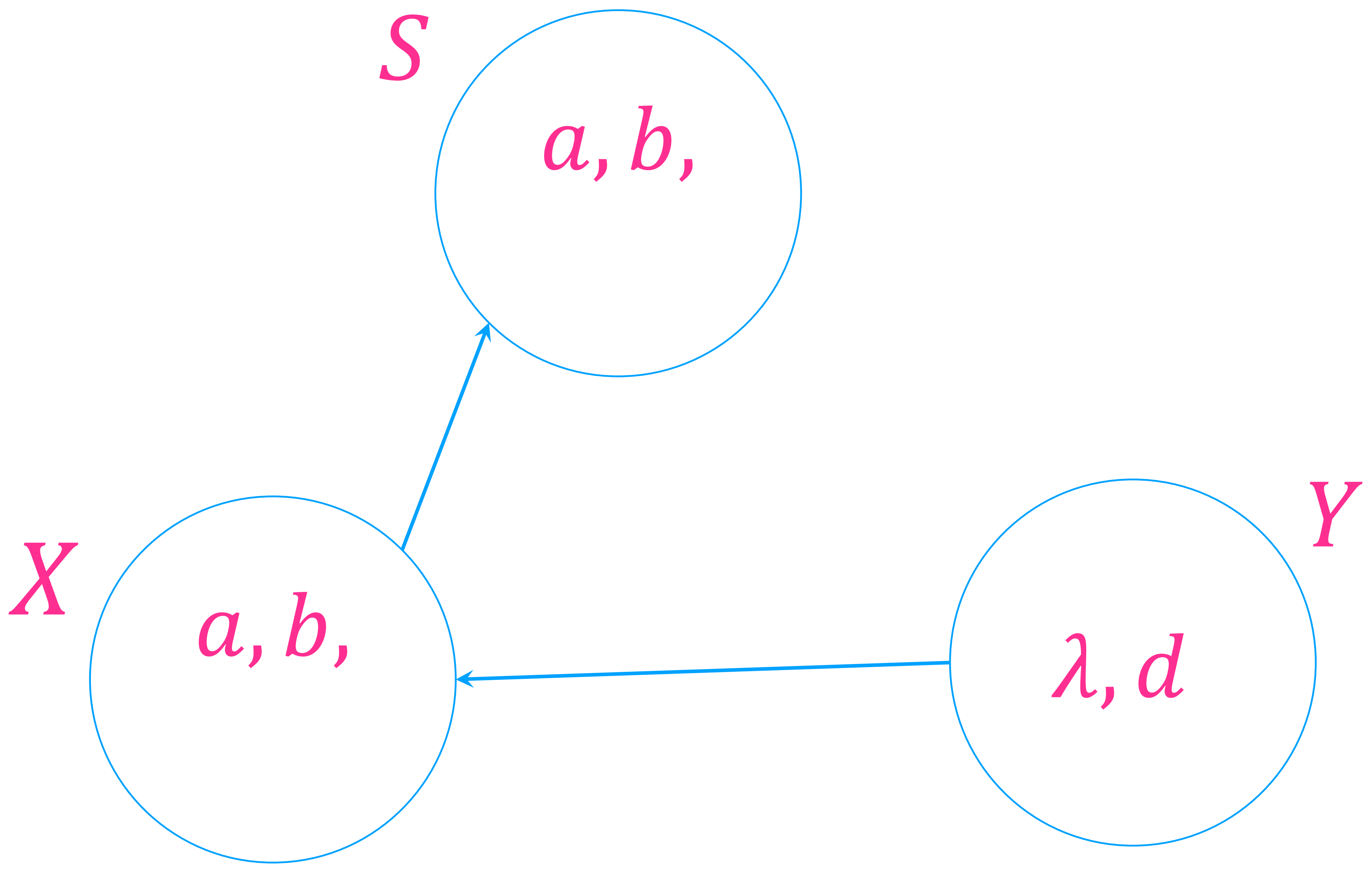

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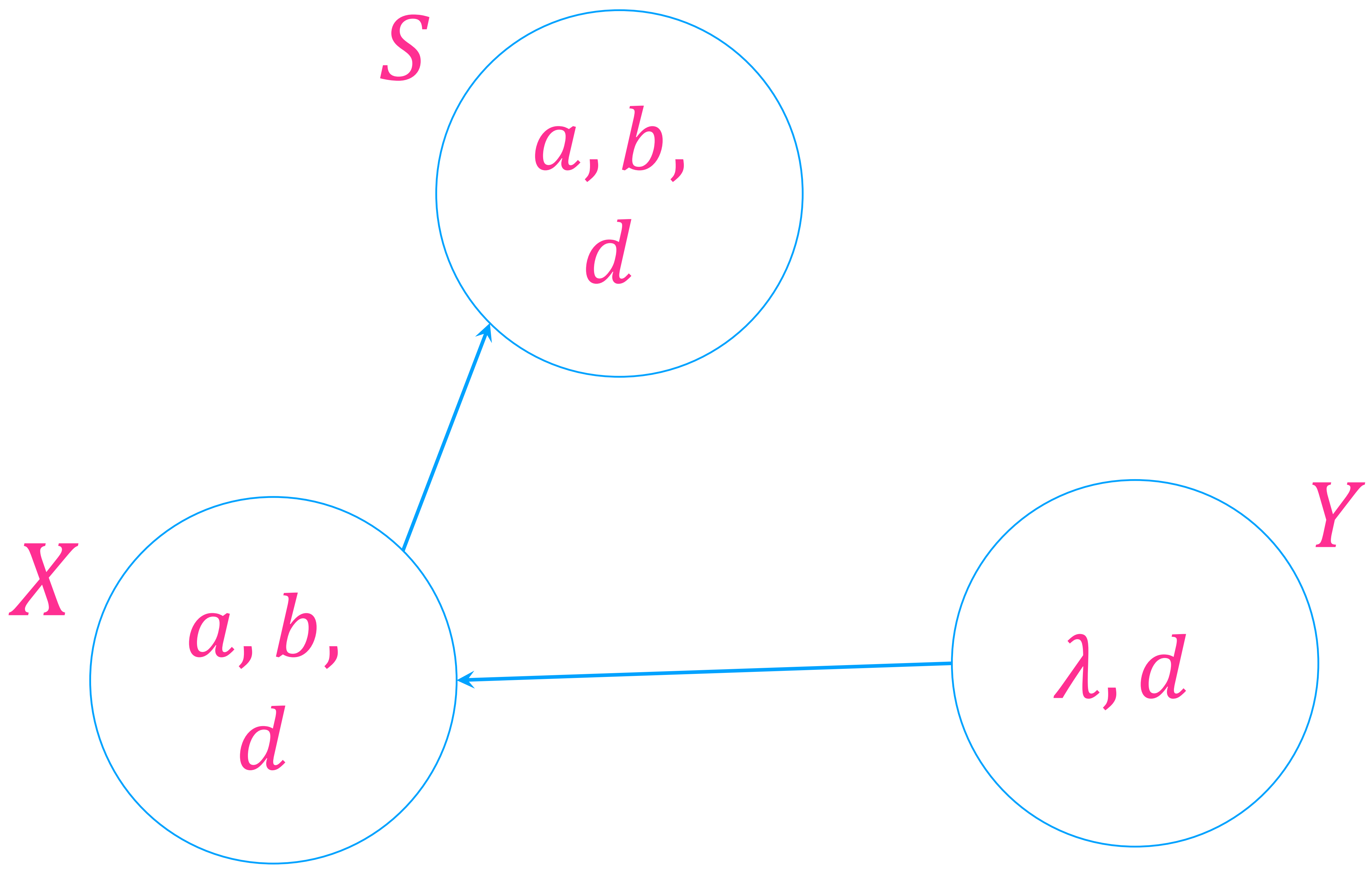

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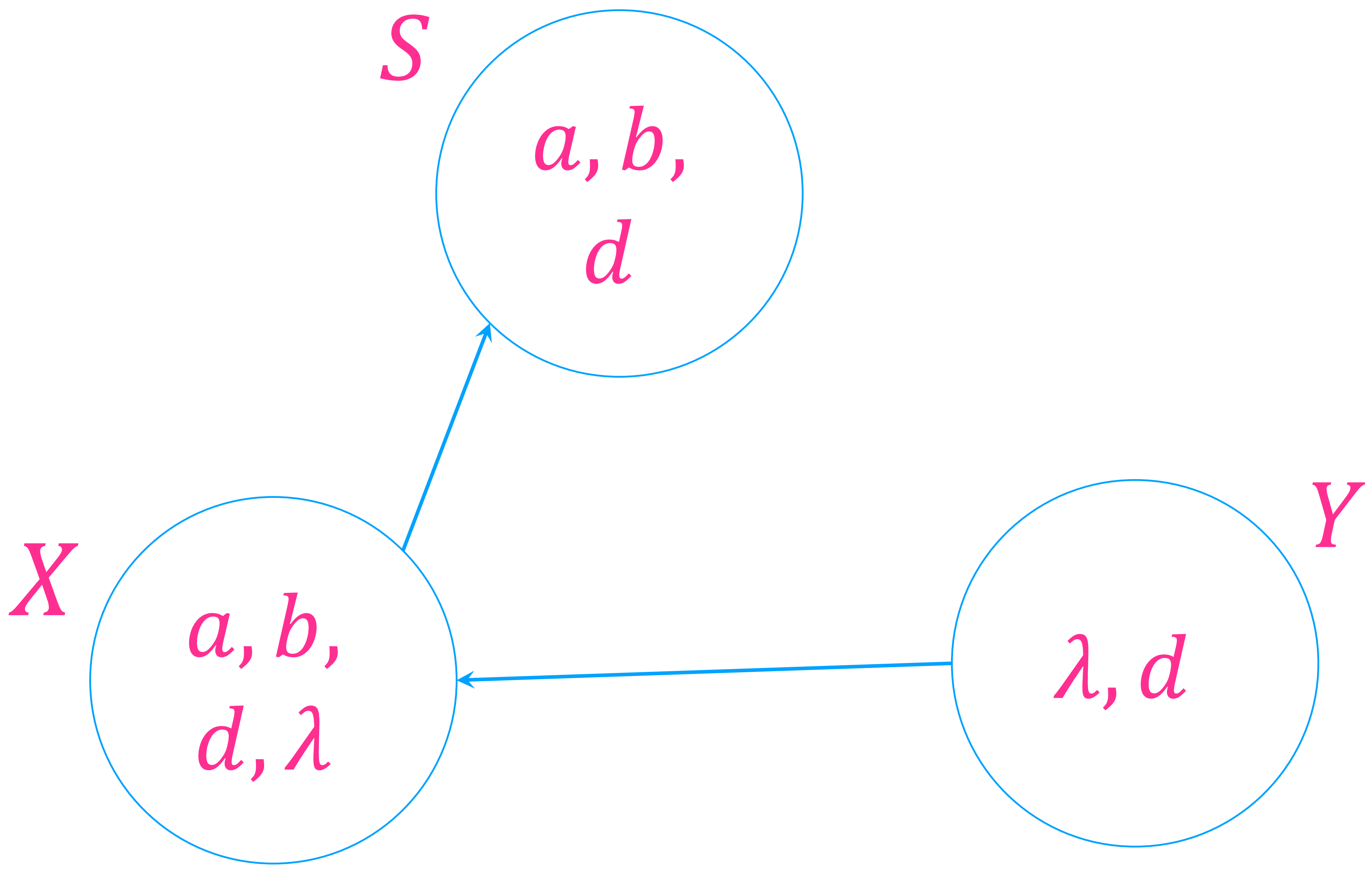

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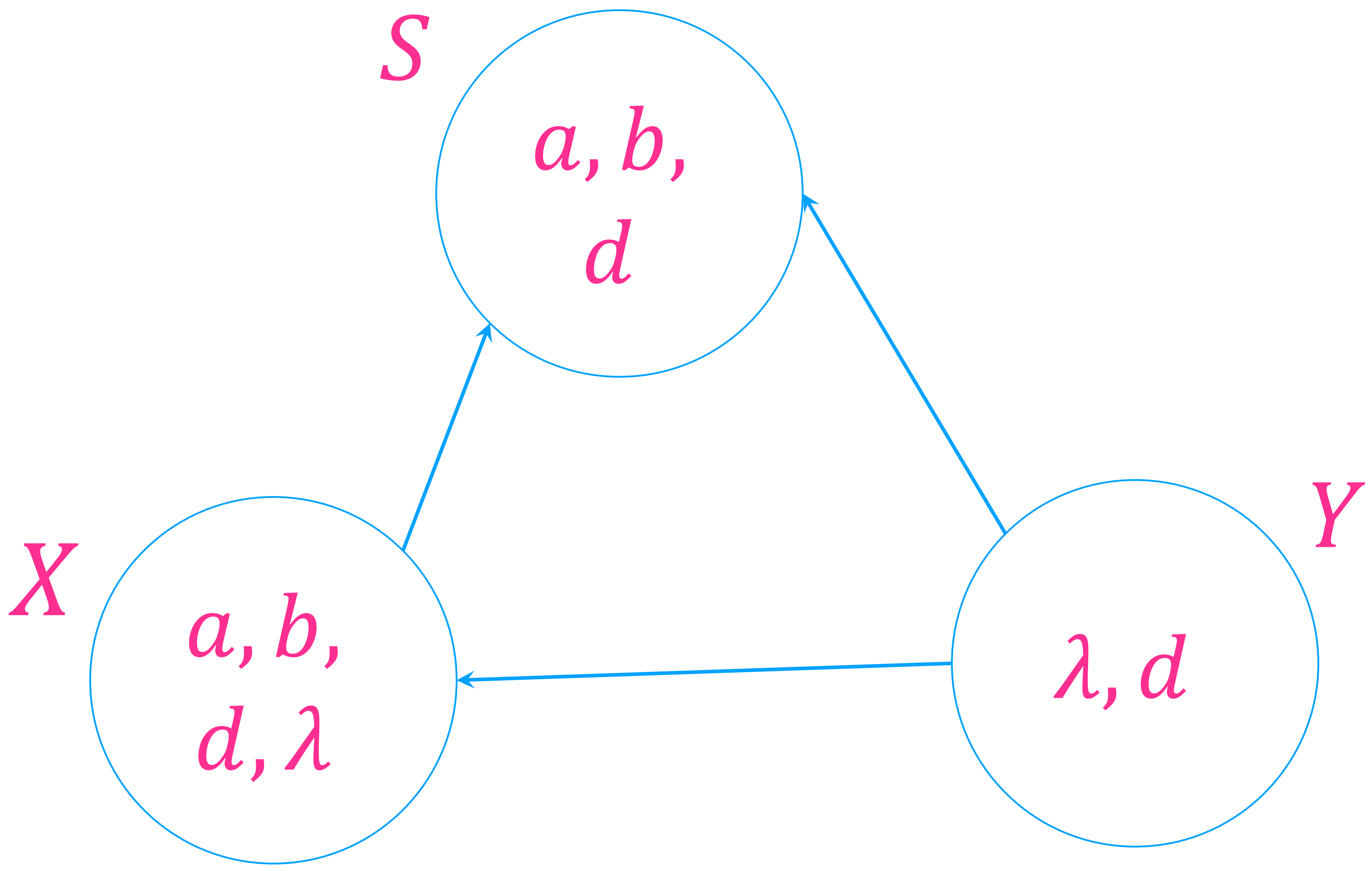

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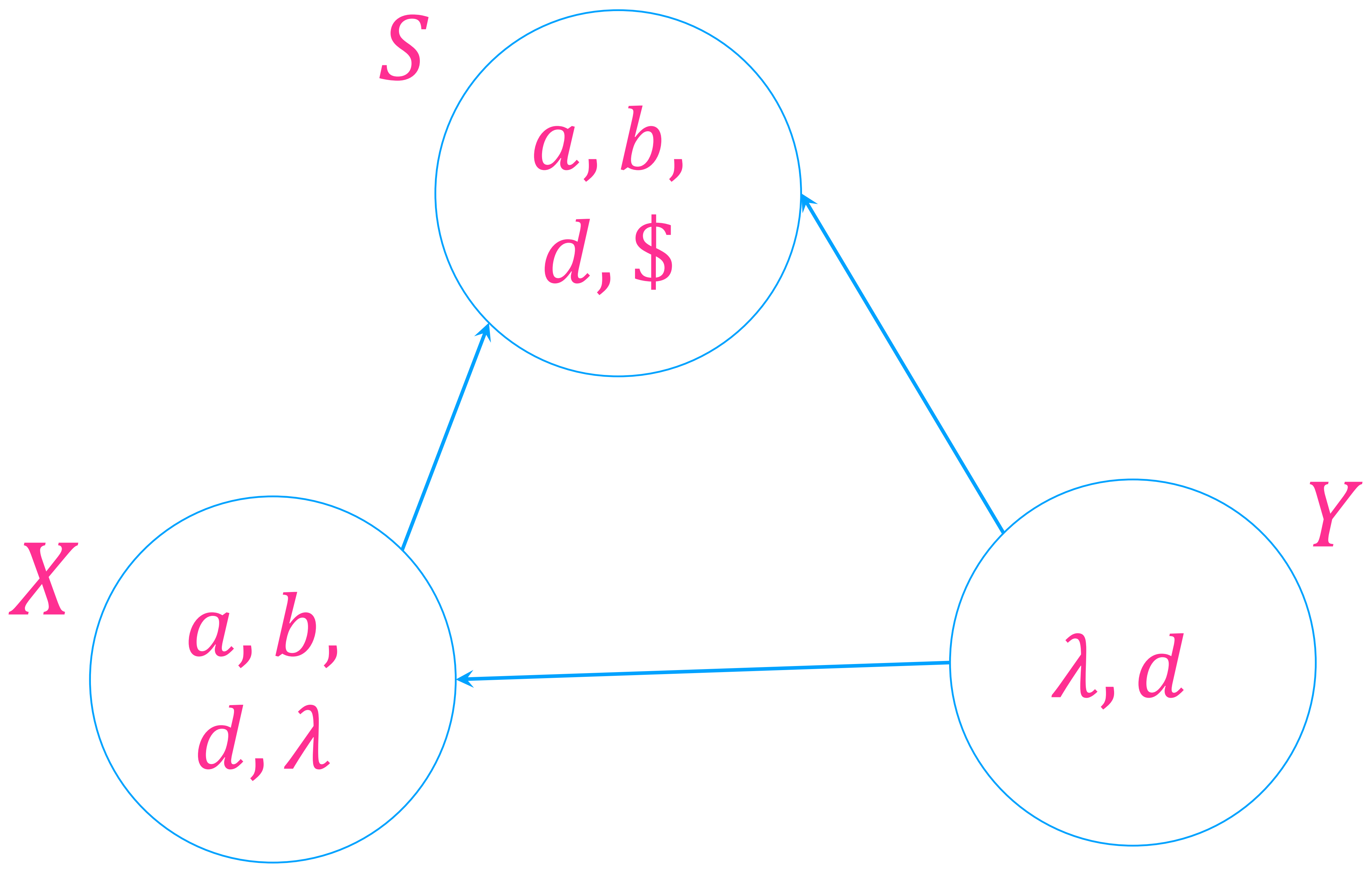

follow sets

- Follow(S) = { }
- To compute Follow(A) :
  - Find productions that have A on the *right hand side*

1. $X \to \alpha A \beta$: Follow(A) $\supseteq$ First(β) - λ
2. $X \to \alpha A \beta$: If $\lambda \in$First(β), Follow(A) $\supseteq$ Follow(X)
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- Note: Follow(X) *never* has λ in it

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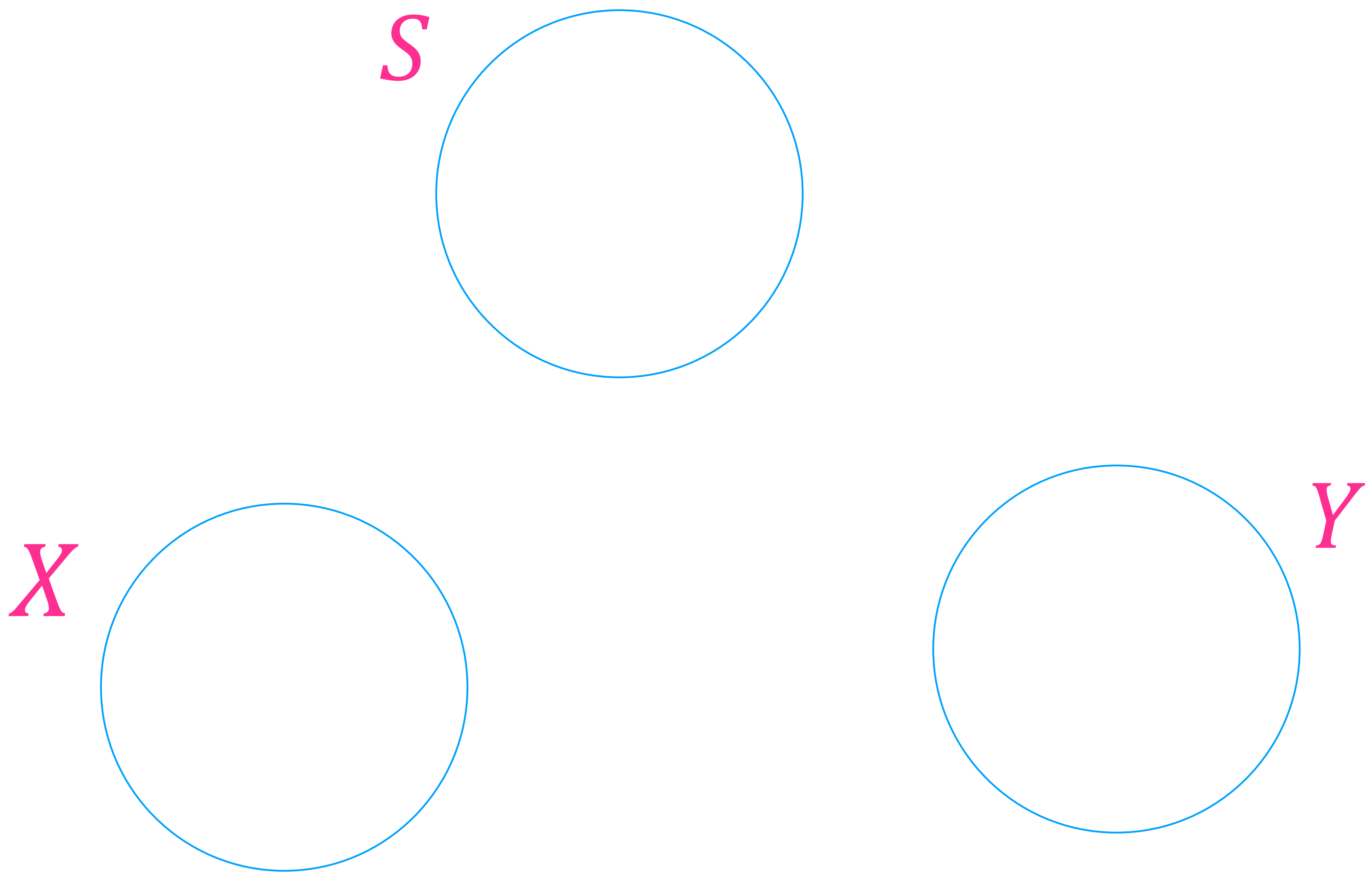

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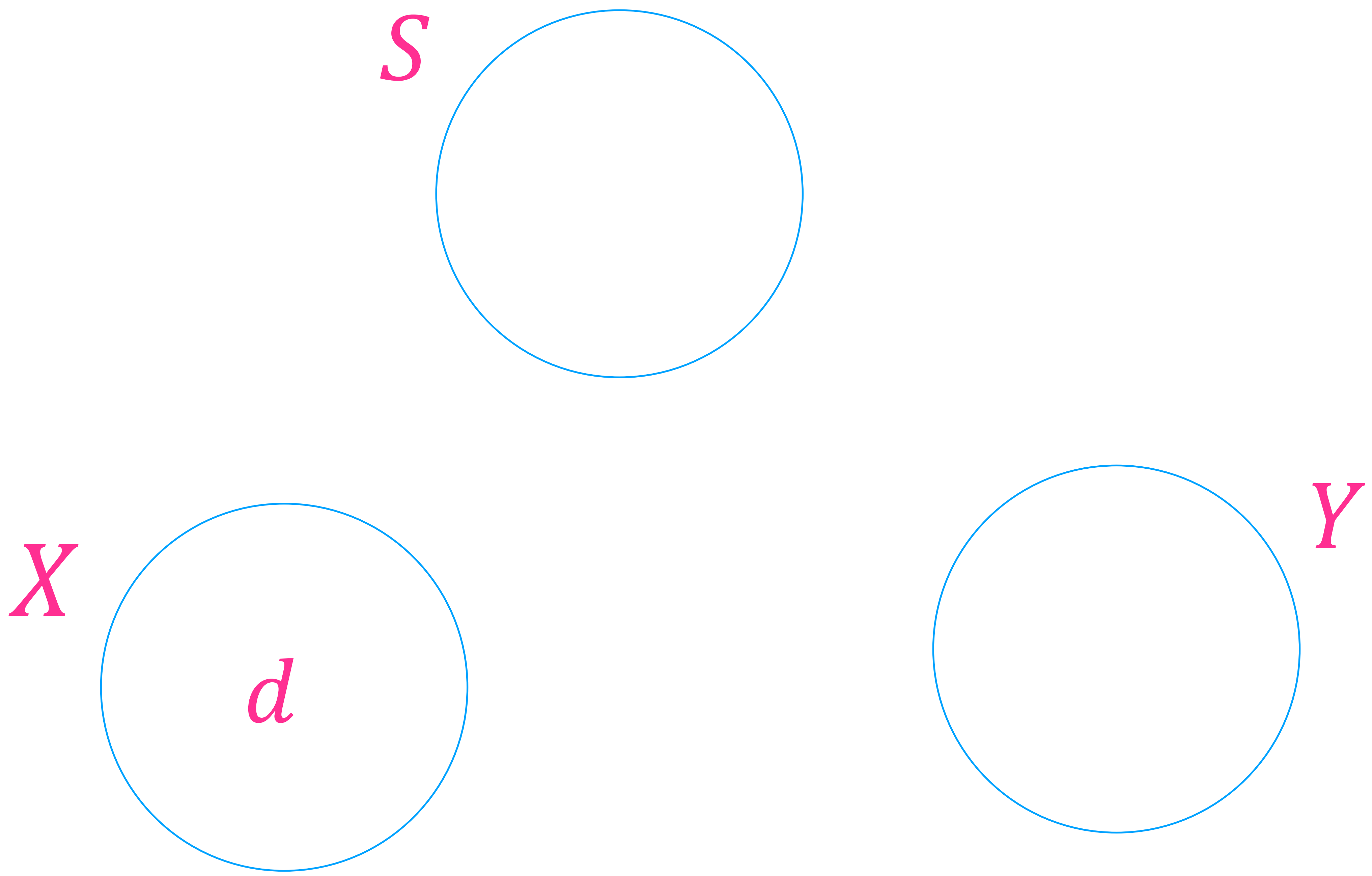

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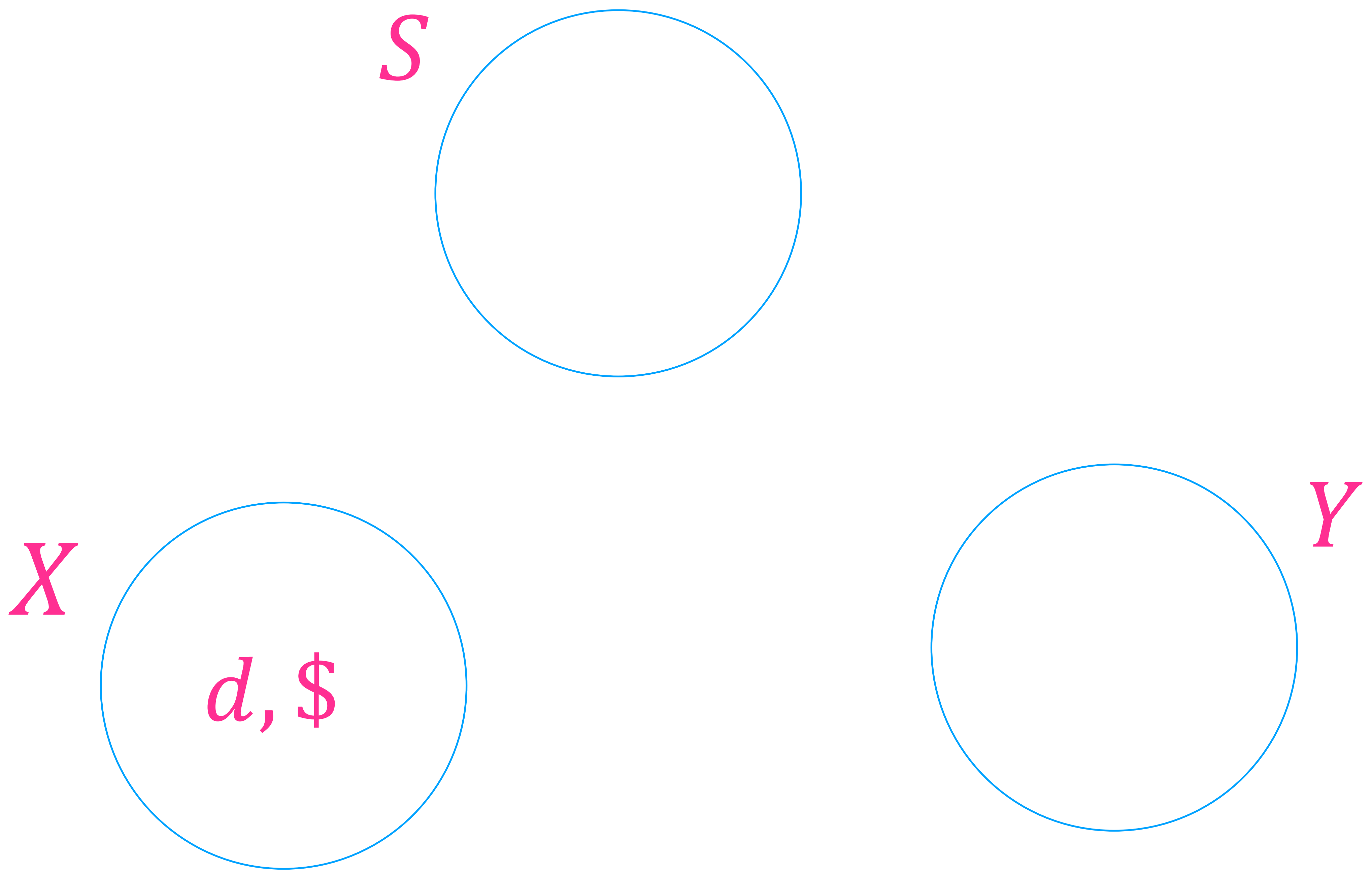

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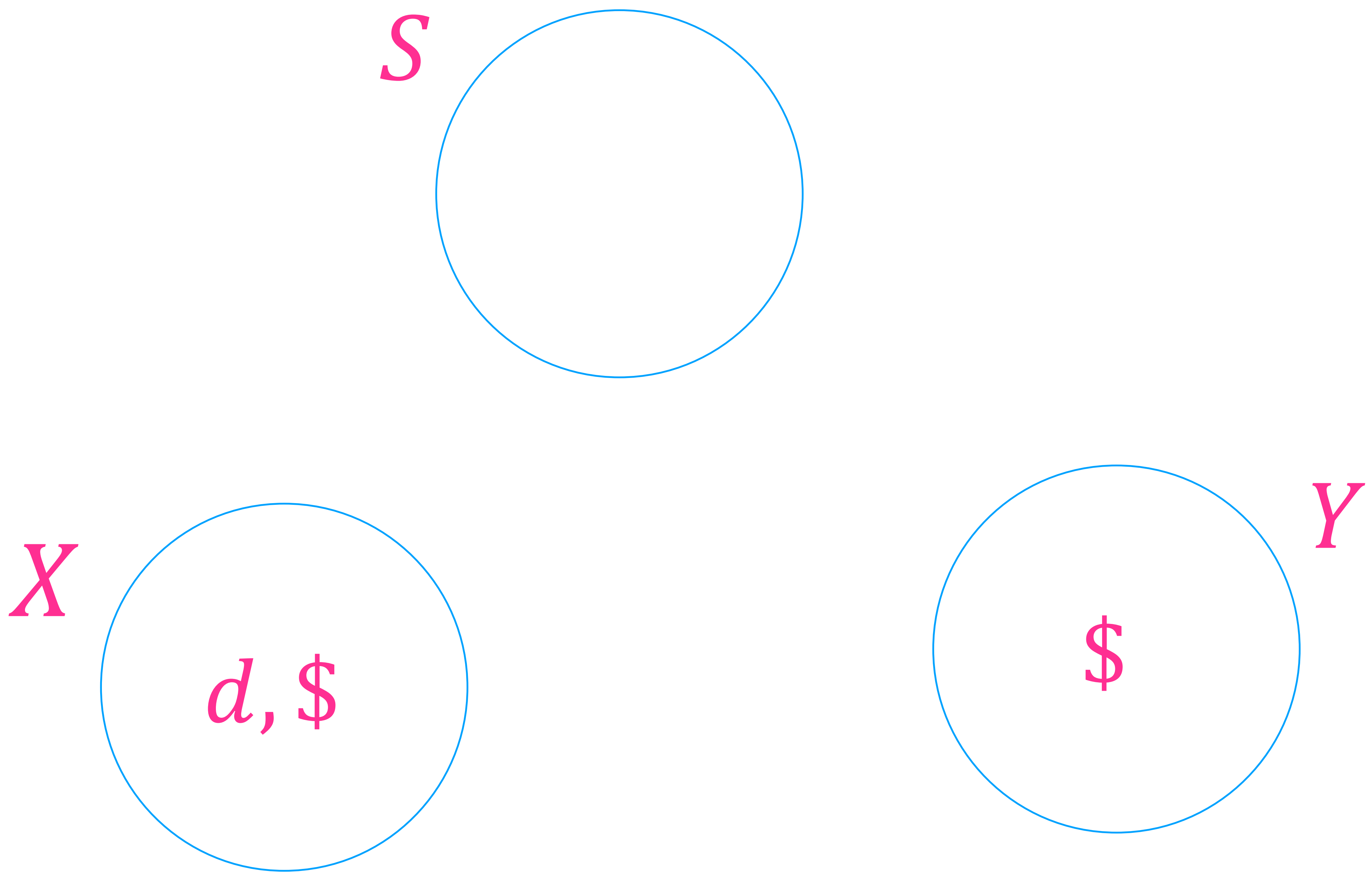

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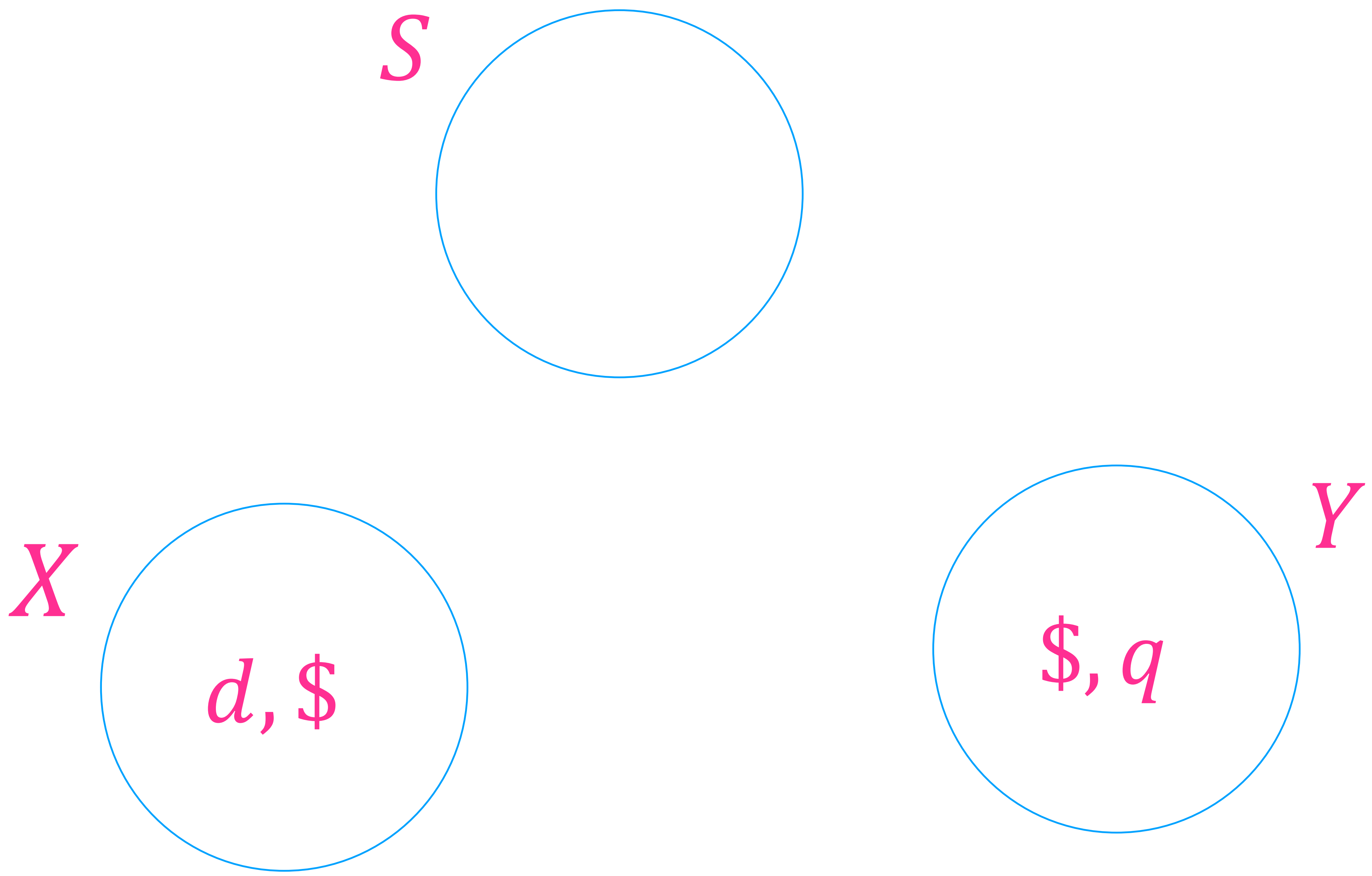

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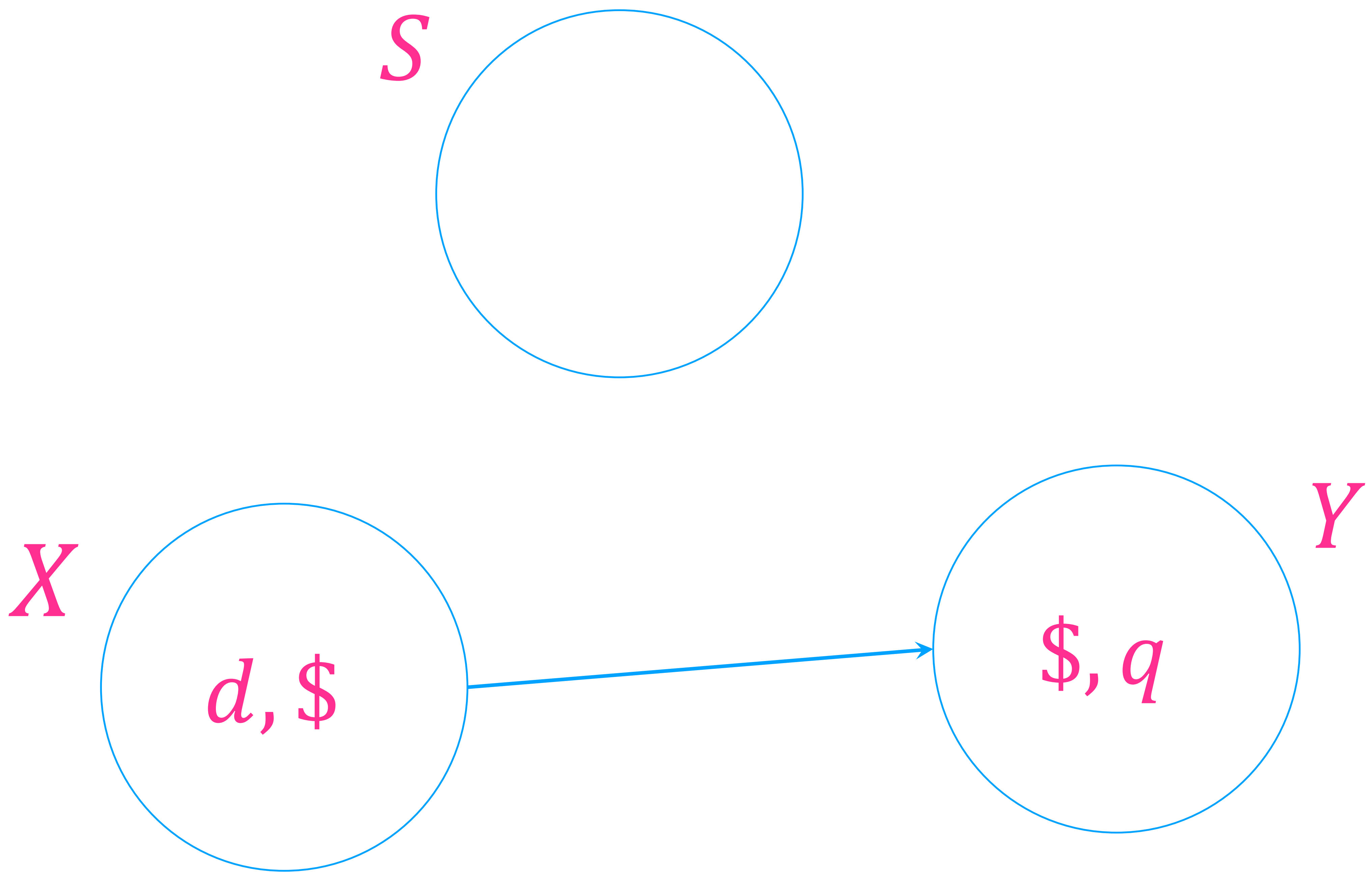

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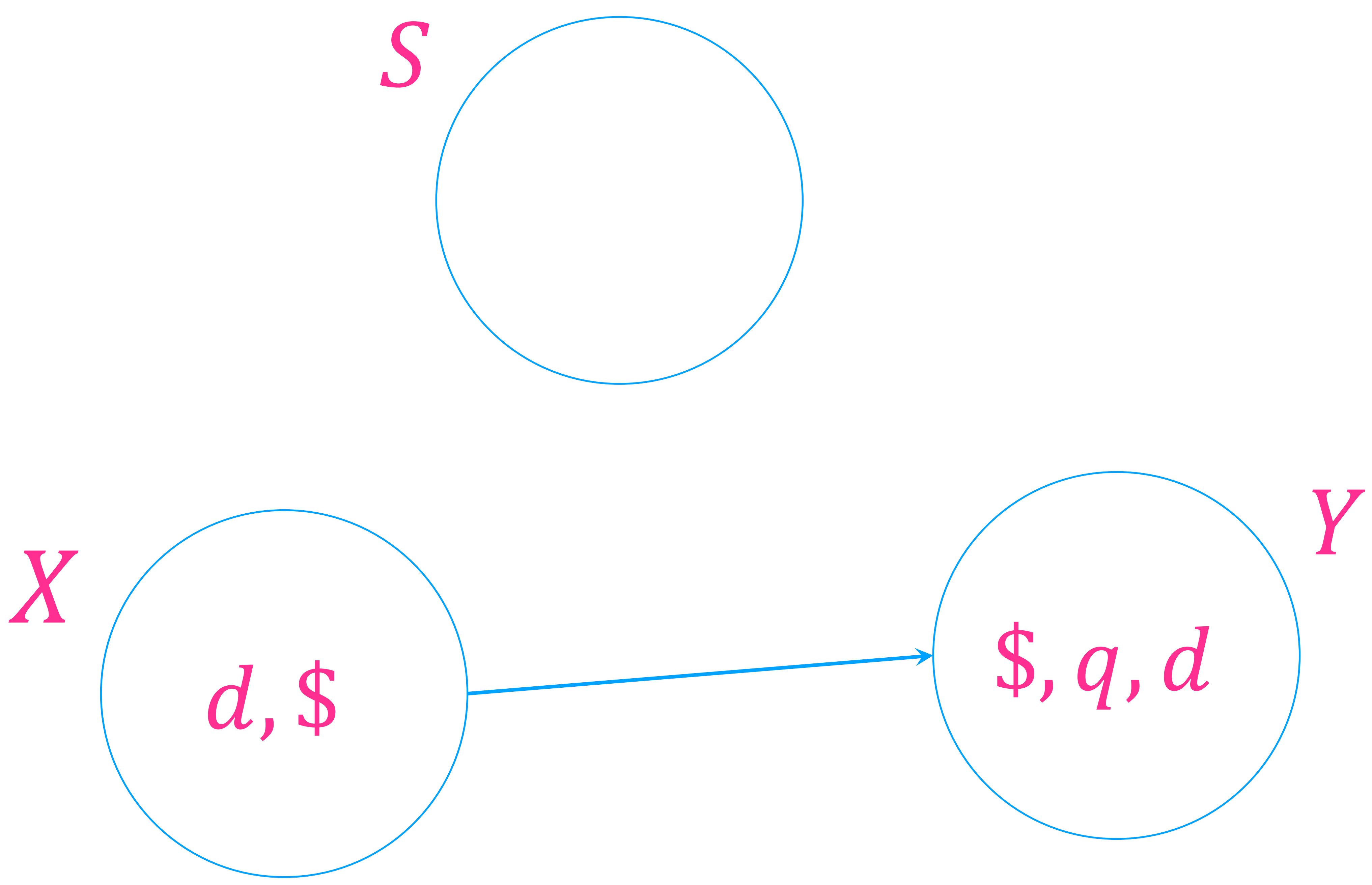

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next: putting it all together