Representing Dependence

Iteration space graphs

- Represent each *dynamic* instance of a loop as a point in a graph
- Draw arrows from one point to another to represent dependences

```
for (i = 0; i < N; i++) {
 a[i + 2] = a[i]
}
```

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- Step 1: Create nodes, 1 for each iteration
  - Note: not 1 for each array location!

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```
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  a[i + 2] = a[i]
}
```
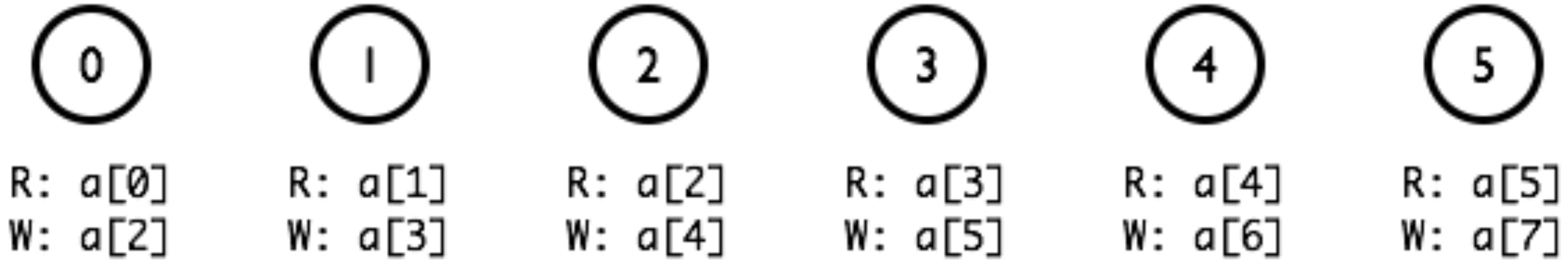

- Step 2: Determine which array elements are read and written in each iteration



Iteration space graphs

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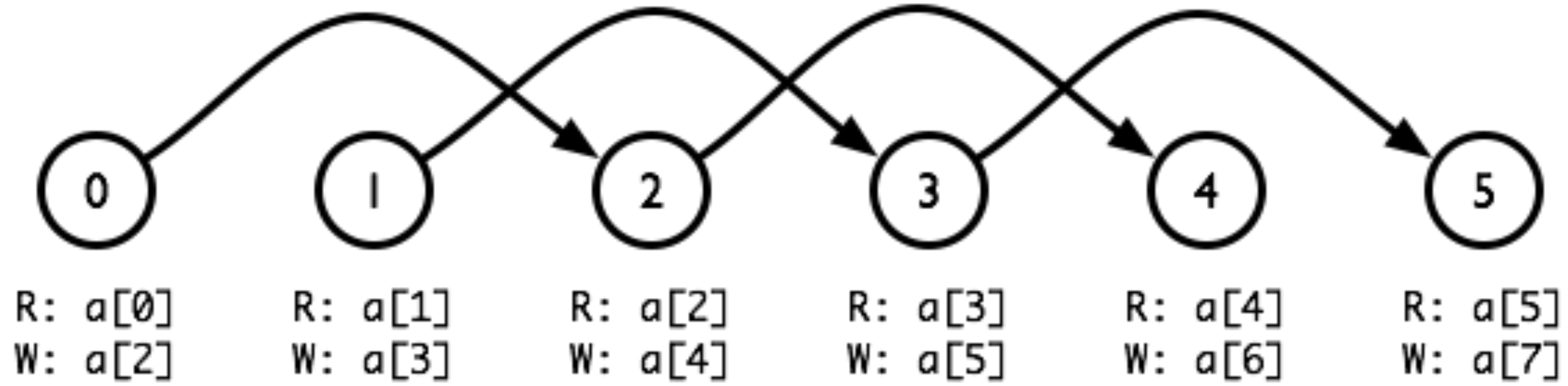
for (i = 0; i < N; i++) {
 a[i + 2] = a[i]
}
```

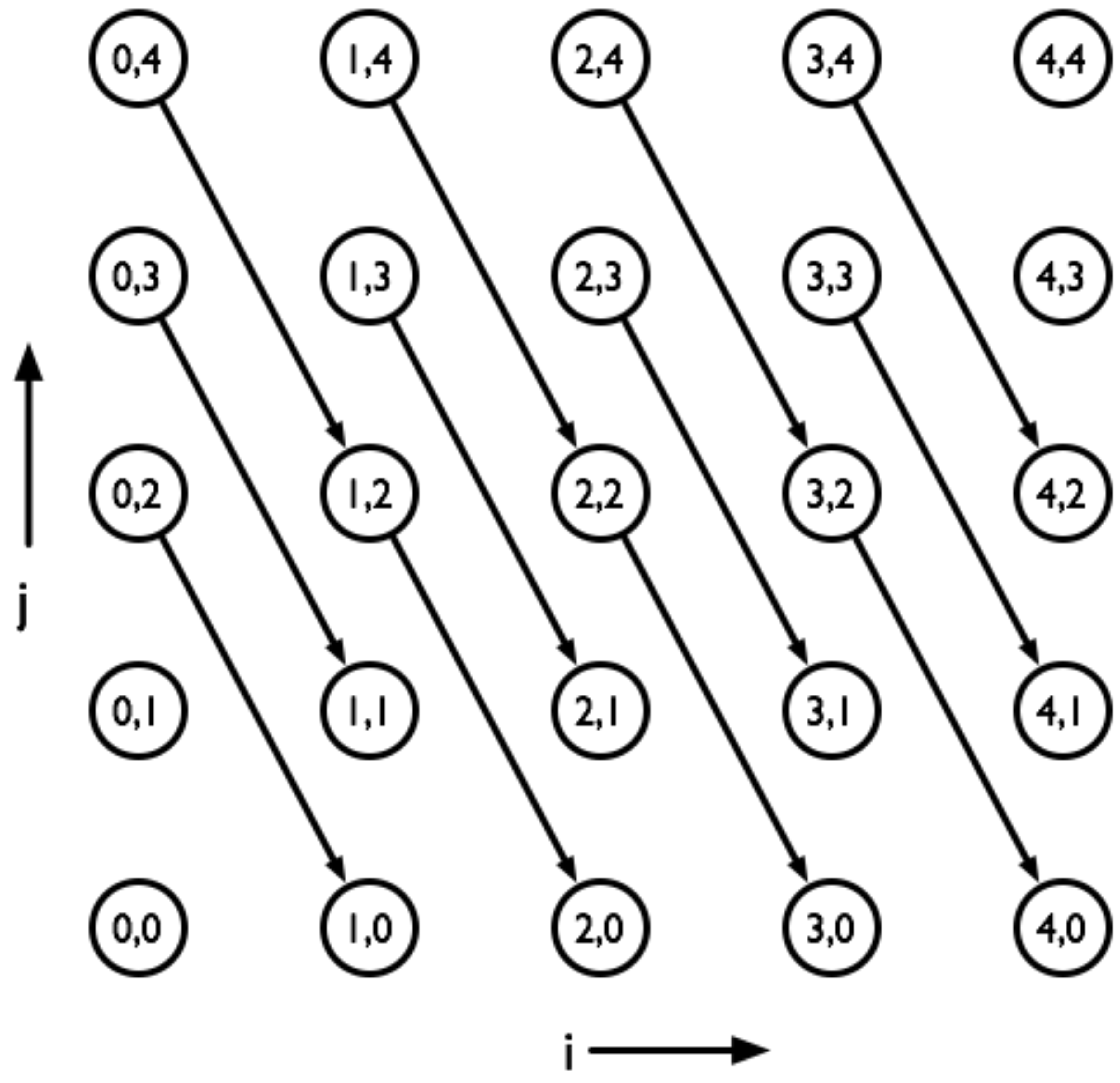
- Step 3: Draw arrows to represent dependences



2-D iteration space graphs

- Can do the same thing for doubly-nested loops
    - 2 loop counters

```
for (i = 0; i < N; i++)
 for (j = 0; j < N; j++)
  …
  a[i+1][j] = a[i][j+2] + 1
  …
```



Iteration space graphs

- Can also represent output and anti dependences
  - Use different kinds of arrows for clarity. *E.g.*
  - for output 
  - for anti 

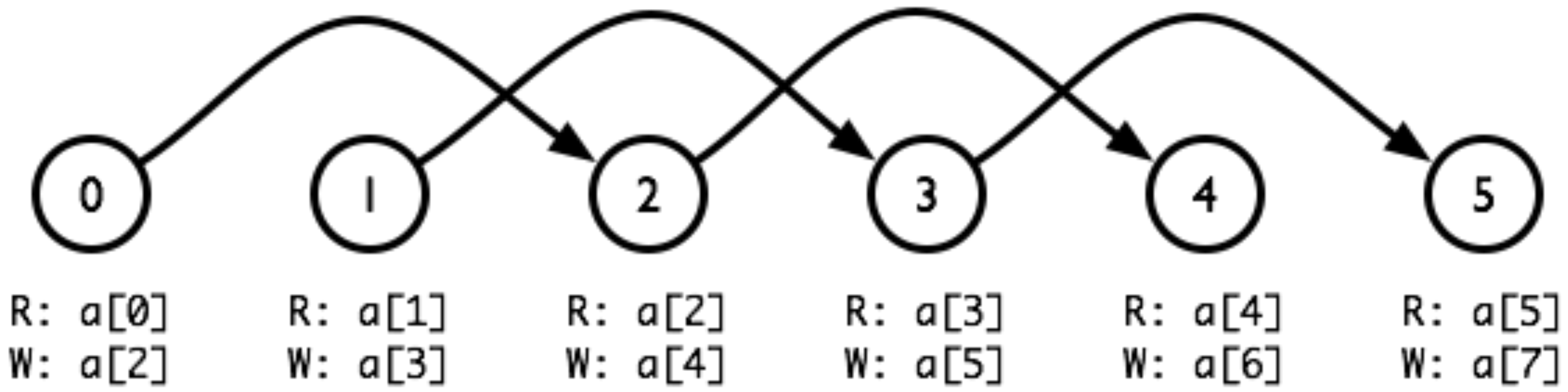
- Crucial problem: Iteration space graphs are potentially infinite representations!
  - Can we represent dependences in a more compact way?

Distance and direction vectors

- Compiler researchers have devised *compressed* representations of dependences
  - Capture the same dependences as an iteration space graph
  - May lose *precision* (show more dependences than the loop actually has)
- Two types
  - **Distance vectors:** captures the "shape" of dependences, but not the particular source and sink
  - **Direction vectors:** captures the "direction" of dependences, but not the particular shape

Distance vector

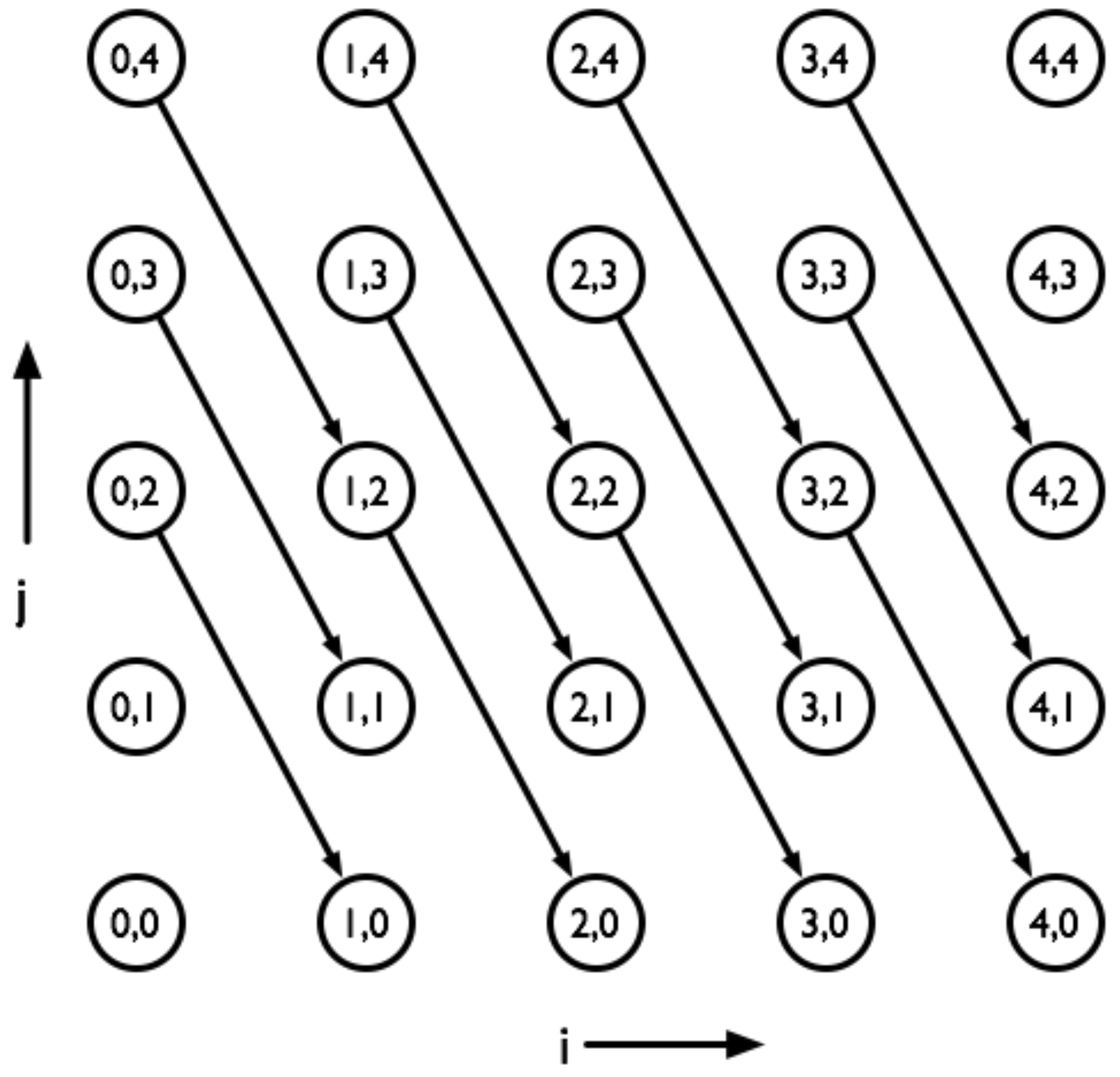
- Represent each dependence arrow in an iteration space graph as a vector
    - Captures the "shape" of the dependence, but loses where the dependence originates



- Distance vector for this iteration space: (2)
    - Each dependence is 2 iterations forward

2-D distance vectors

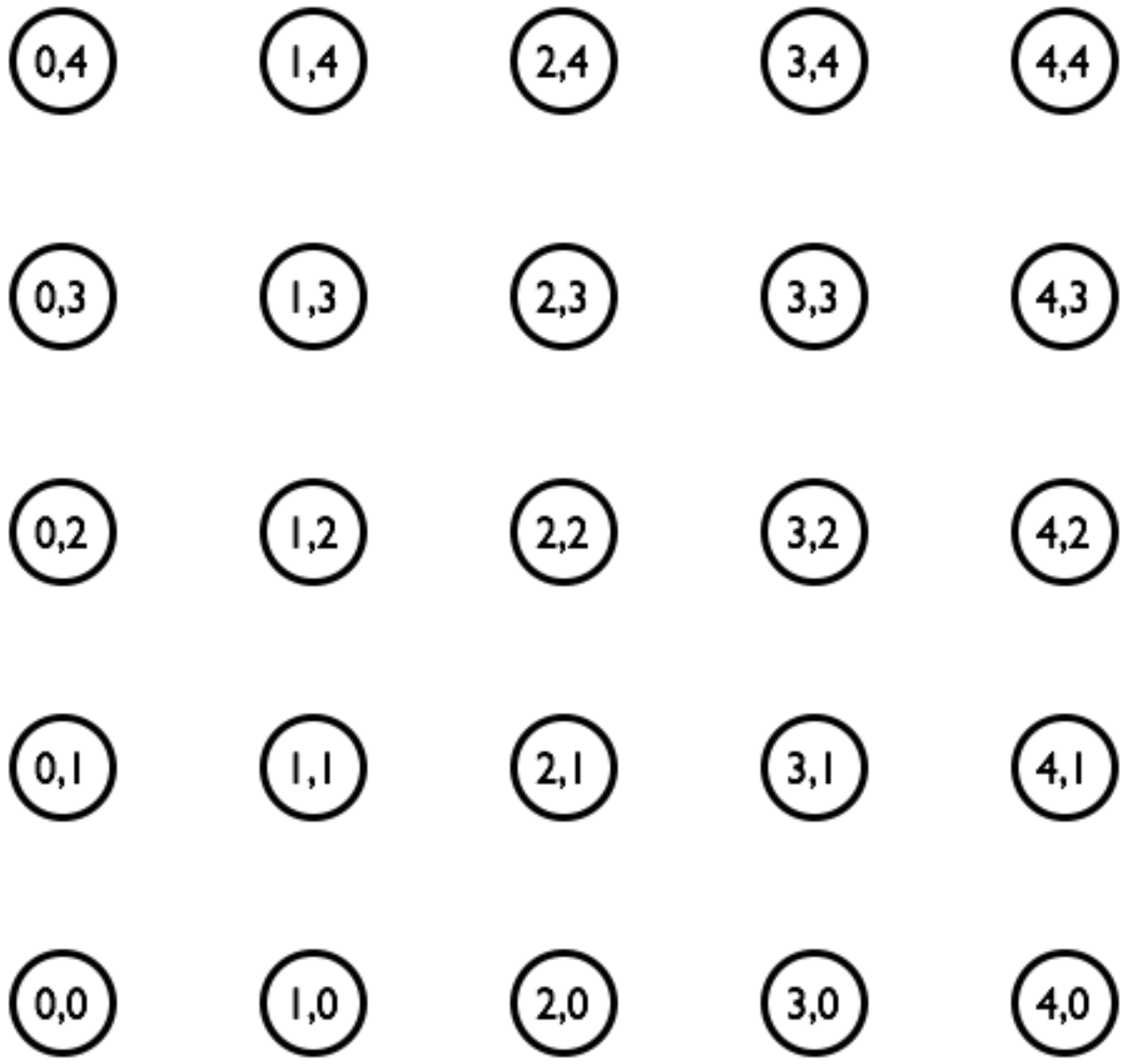
- Distance vector for this graph:
  - (1, -2)
  - +1 in the i direction, -2 in the j direction
- Crucial point about distance vectors: they are always "positive"
  - First non-zero entry has to be positive
  - Dependences can't go backwards in time



More complex example

- Can have multiple distance vectors

```
for (i = 0; i < N; i++)
 for (j = 0; j < N; j++)
  a[i+2][j] = a[i+1][j+2] + a[i][j]
```



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 for (j = 0; j < N; j++)
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```
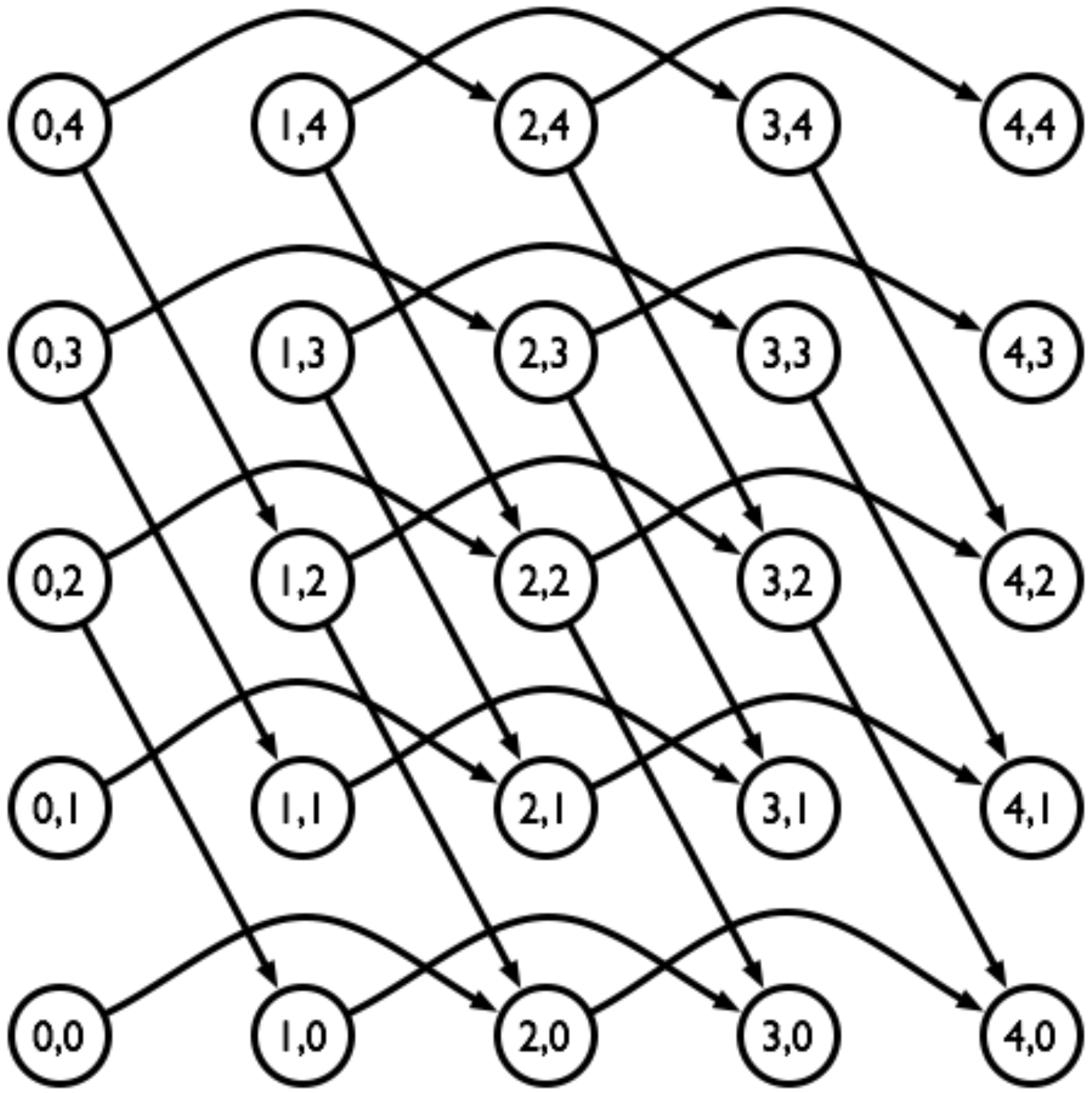

- Distance vectors
  - (1, -2)
  - (2, 0)
- Important point: order of vectors depends on order of loops, not use in arrays



Problems with distance vectors

- The preceding examples show how distance vectors can precisely summarize all the dependences in a loop nest using just a small number of distance vectors
- Can't always summarize as easily!
- Running example:

```
for (i = 0; i < N; i++)
 a[2*i] = a[i];
```

Loss of precision

- What are the distance vectors for this code?
  - (1), (2), (3), (4) ...
- Note: we have information about the length of each vector, but not about the source of each vector
  - What happens if we try to reconstruct the iteration space graph?

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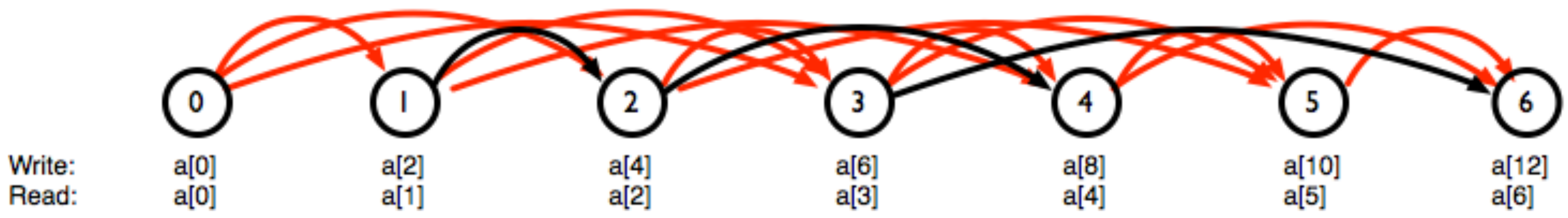

Direction vectors

- The whole point of distance vectors is that we want to be able to succinctly capture the dependences in a loop nest
    - But in the previous example, not only did we add a lot of extra information, we still had an infinite number of distance vectors
- Idea: summarize distance vectors, and save only the *direction* the dependence was in
    - (2, -1) → (+, –)
    - (0, 1) → (0, +)
    - (0, -2) → (0, –) *(can’t happen; dependences have to be positive)*
    - Notation: sometimes use ‘<‘ and ‘>’ instead of ‘+’ and ‘–’

Why use direction vectors?

- Direction vectors lose a lot of information, but do capture some useful information
  - Whether there is a dependence (anything other than a ‘0’ means there is a dependence)
  - Which dimension and direction the dependence is in
- Many times, the only information we need to determine if an optimization is legal is captured by direction vectors
  - Loop parallelization
  - Loop interchange