Functions over Domains

Functions on domains

- If D is a domain, we can define a function $f : D \to D$
  - Function maps each element of domain on to another element of the domain
- Example: for D = powerset of {a, b, c}
  - $f(x) = x \cup \{a\}$
  - $g(x) = x - \{a\}$
  - $h(x) = \{a\} - x$

Monotonic functions

- A function $f : D \to D$ on a domain D is *monotonic* if
  - $x \sqsubseteq y \Rightarrow f(x) \sqsubseteq f(y)$
- Note: this is not the same as $x \sqsubseteq f(x)$
  - This means that f is *extensive*
- Intuition: think of f as an electrical circuit mapping input to output
  - If f is monotonic, raising the input voltage raises the output voltage (or keeps it the same)
  - If f is extensive, the output voltage is always the same or more than the input voltage

Examples

- Domain D is the powerset of {a, b, c}
- Montonic functions:
  - f(x) = { } (why?)
  - f(x) = x ∪ {a}
  - f(x) = x - {a}
- Not monotonic
  - f(x) = {a} - x (why?)

- Extensivity
  - f(x) = x ∪ {a} **is monotonic *and* extensive**
  - f(x) = x - {a} **is monotonic but not extensive**
  - f(x) = {a} - x **is neither**
- What is a function that is extensive, but not monotonic?

Fixpoints

- Suppose $f : D \rightarrow D$.
  - A value x is a *fixpoint* of f if $f(x) = x$
  - f maps x to itself

- Examples: D is a powerset of {a, b, c}
  - Identity function: f(x) = x
    - Every element is a fixpoint
  - f(x) = x ∪ {a}
    - Every set that contains a is a fixpoint
  - f(x) = {a} - x
    - No fixpoints

next: fixpoint theorem