Loops and Fixpoints

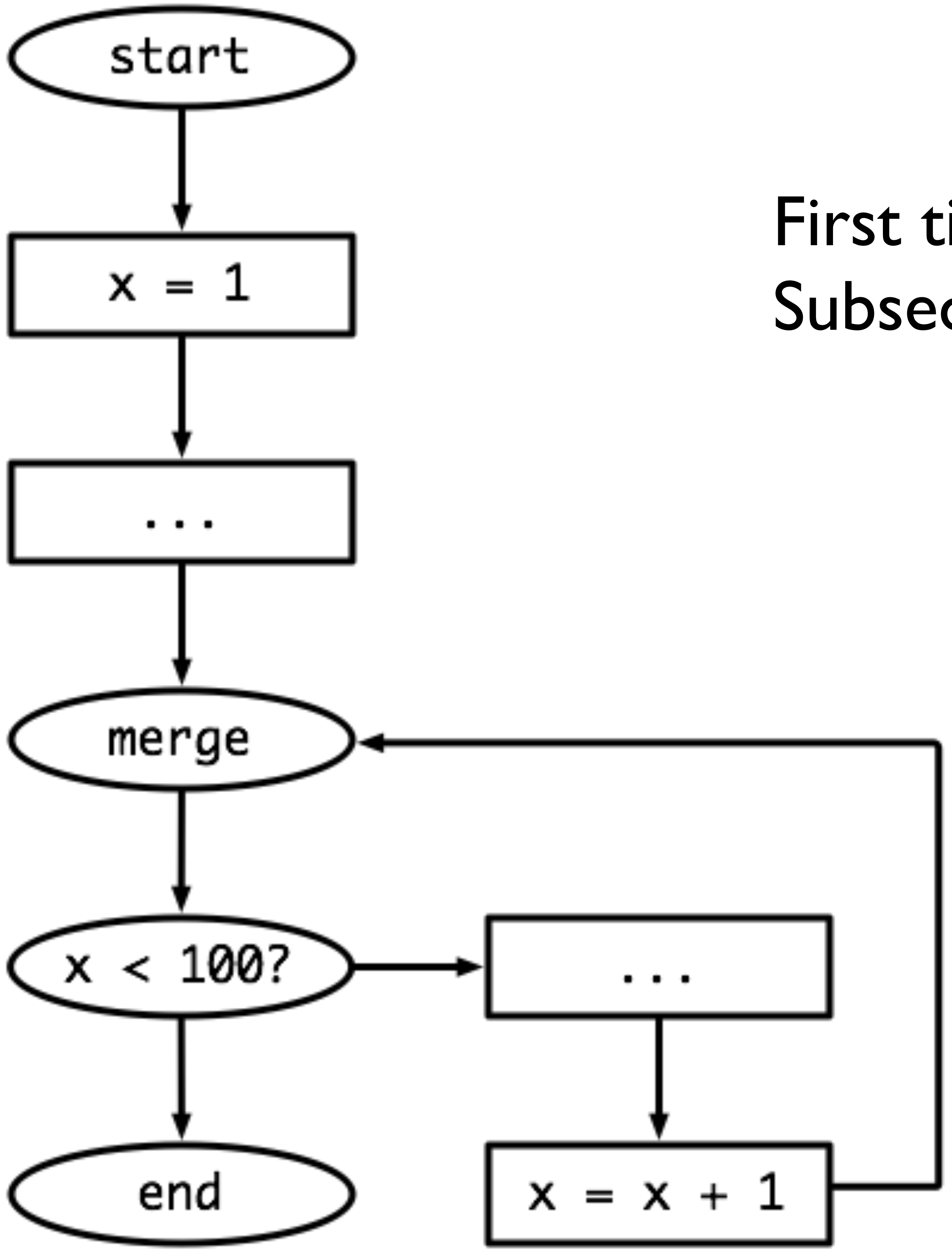
what about loops?

- Symbolically execute each statement in the program

- Treat loops as a **fixpoint** problem
    - If the inputs to a statement change, re-execute statement
    - Keep going until inputs stop changing

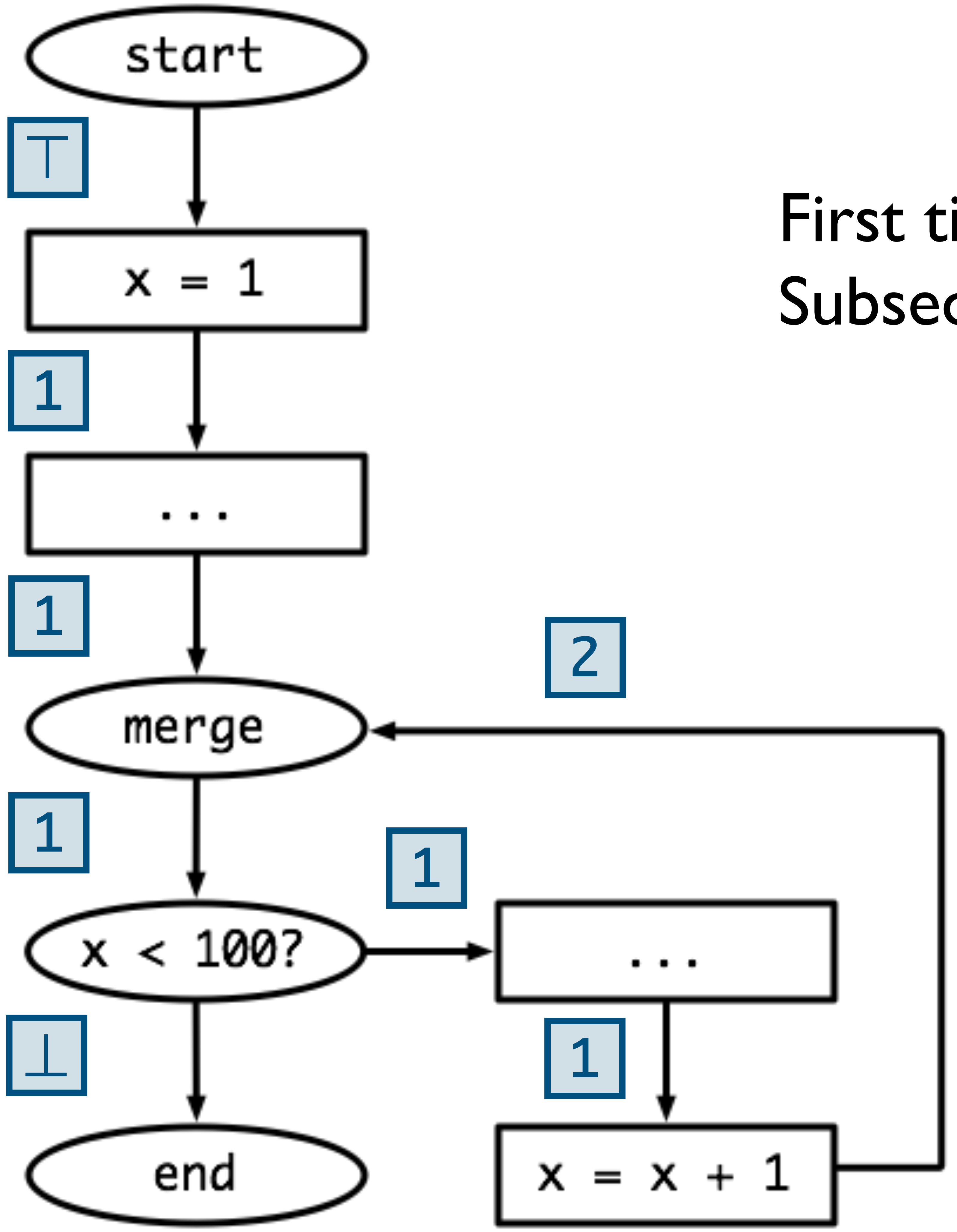
- Claim: this will handle loops
- Claim: inputs will eventually stop changing

loop example



First time through loop, x = 1
Subsequent times, x = ⊤

loop example



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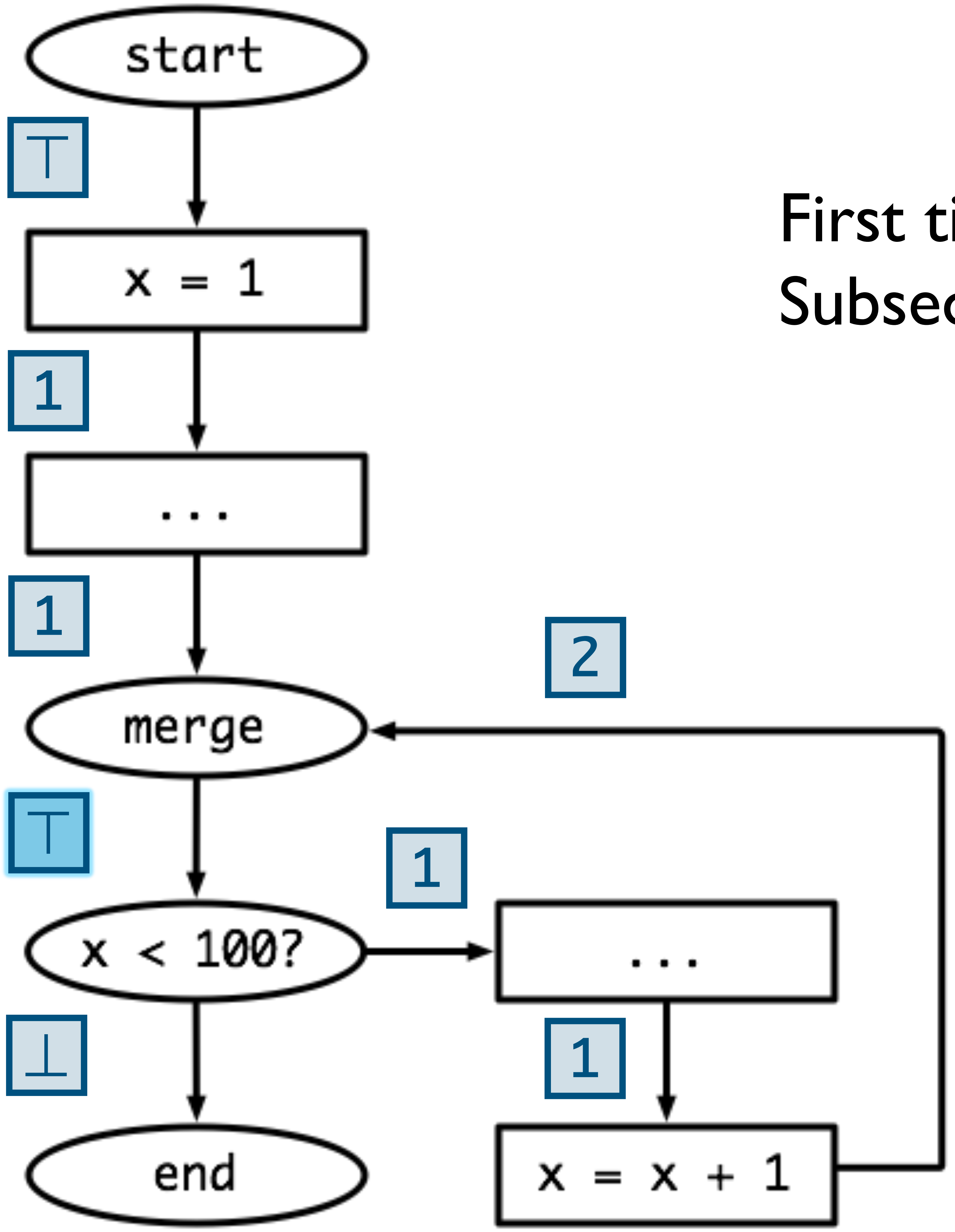
loop example



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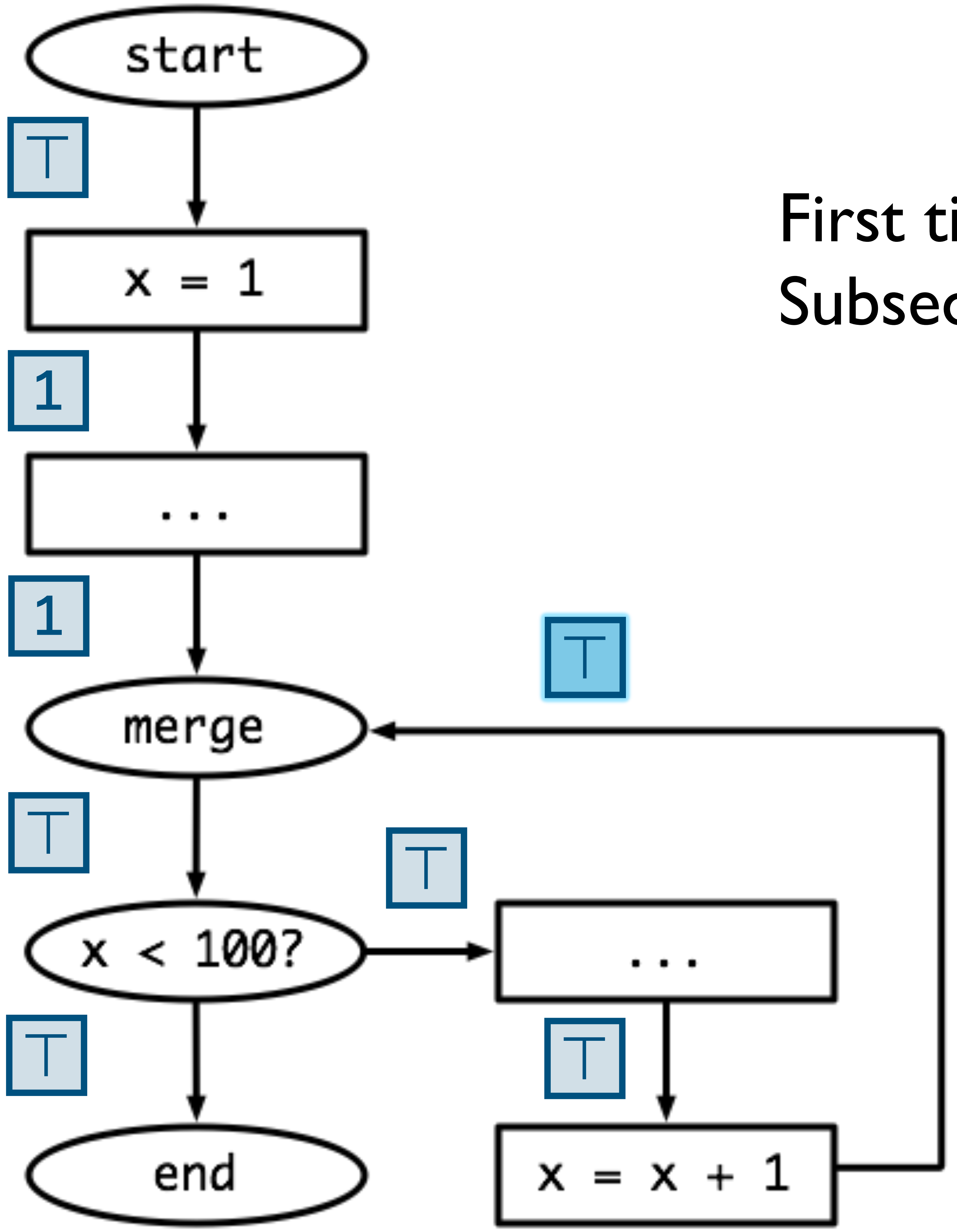
loop example



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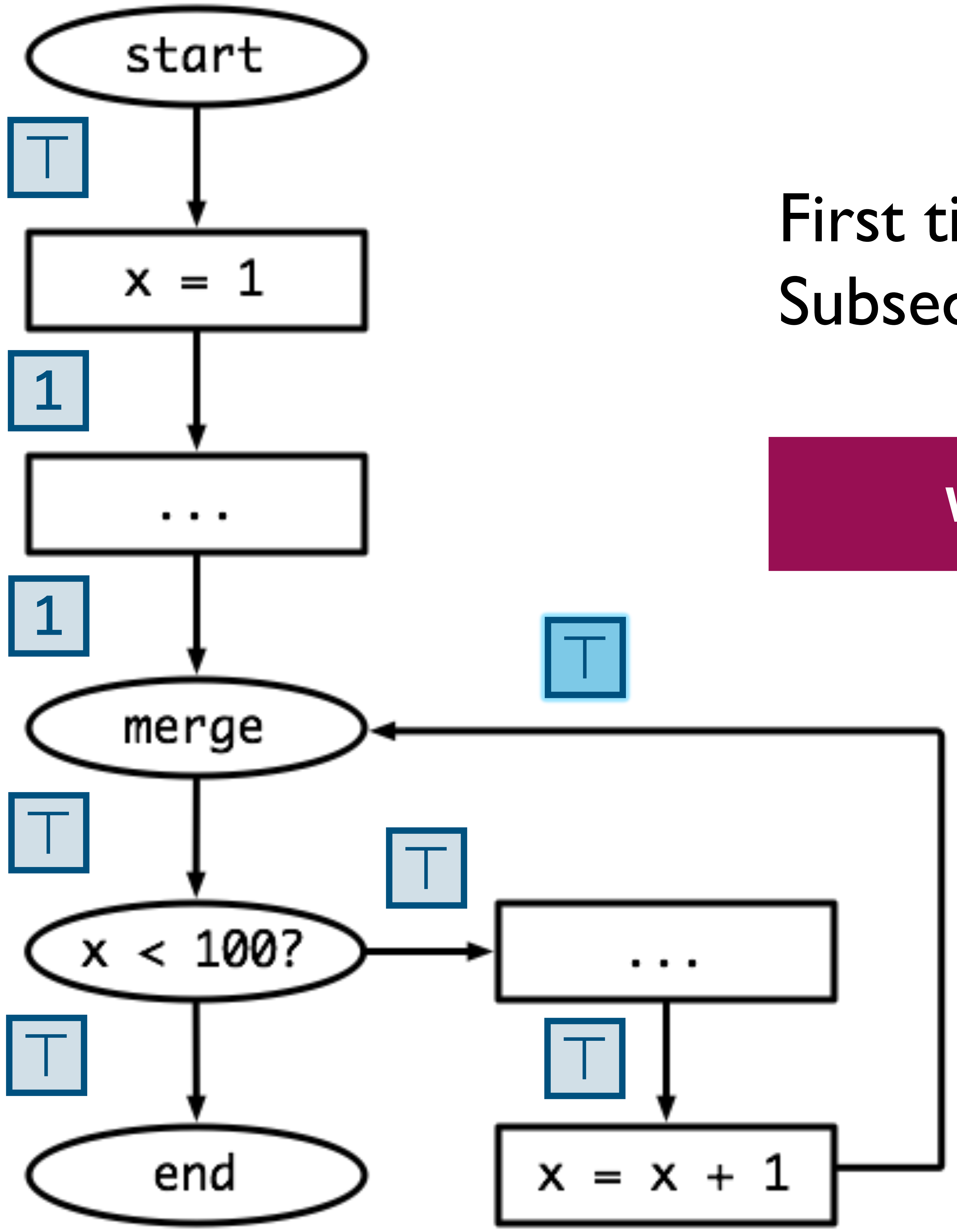
loop example



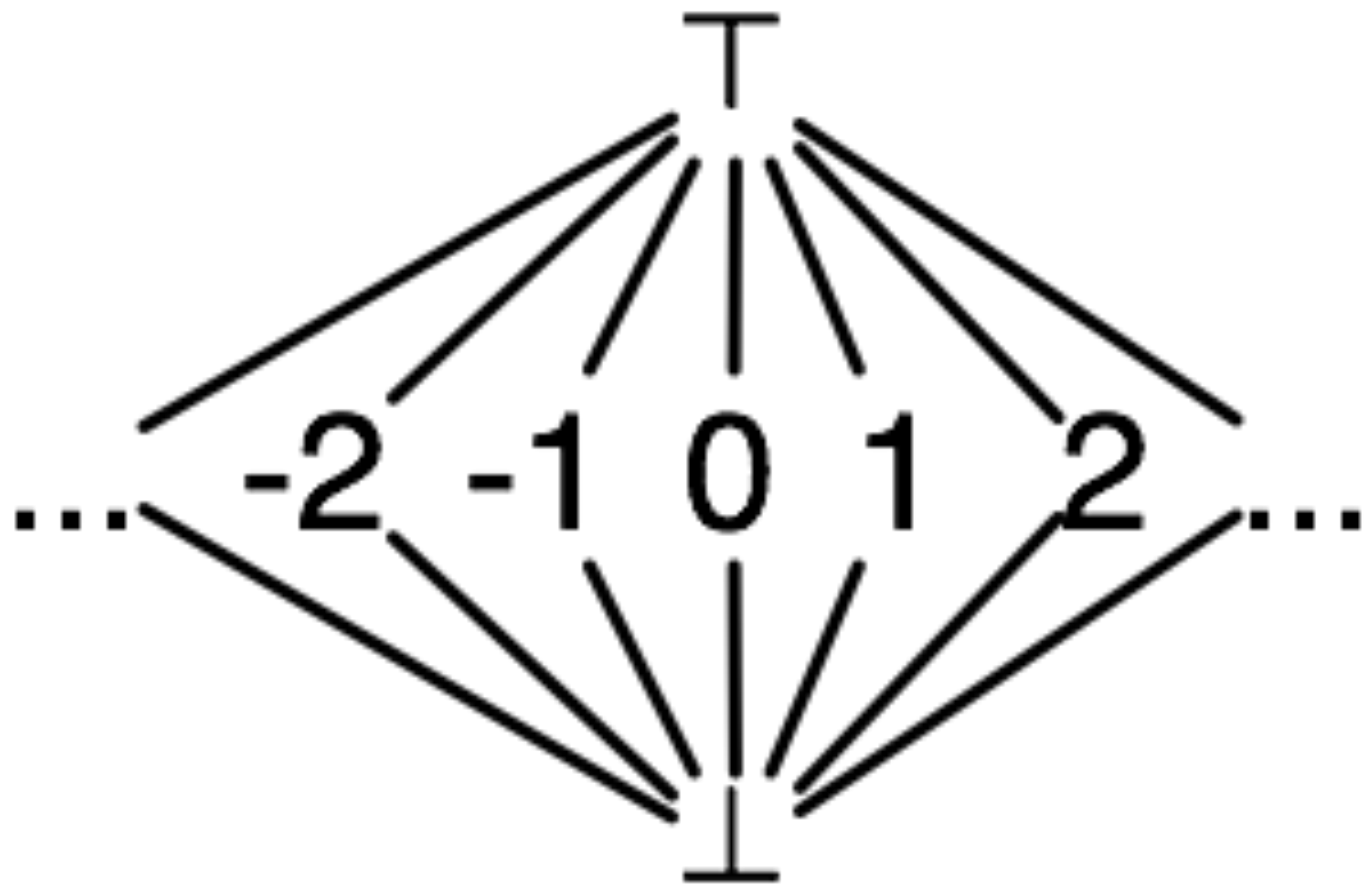
First time through loop, x = 1
Subsequent times, x = ⊤

Why does this work?

lattices

- Symbolic values during execution can be organized according to "amount of information" in a **lattice**

- $\top$ has more information than any constant; any constant has more information than $\perp$



merge in lattices

- Rules for merging basically say merge the information coming from the two branches: “find the *smallest* symbol that has *at least as much* information as the two symbols”

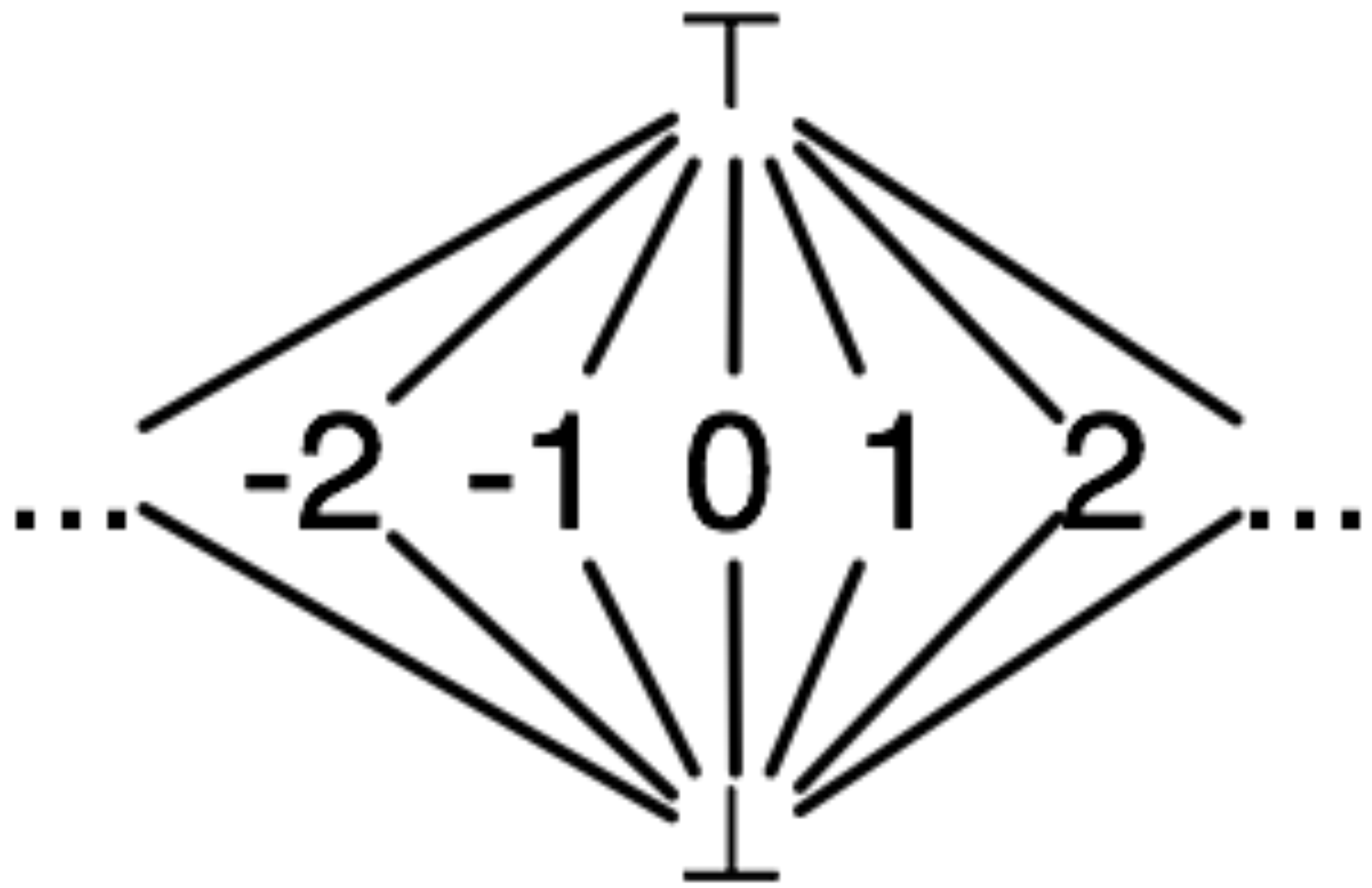
- Special symbol for this **join** operation: $\sqcup$

1. $v_1 \sqcup v_1 \rightarrow v_1$

2. $\top \sqcup * \rightarrow \top$

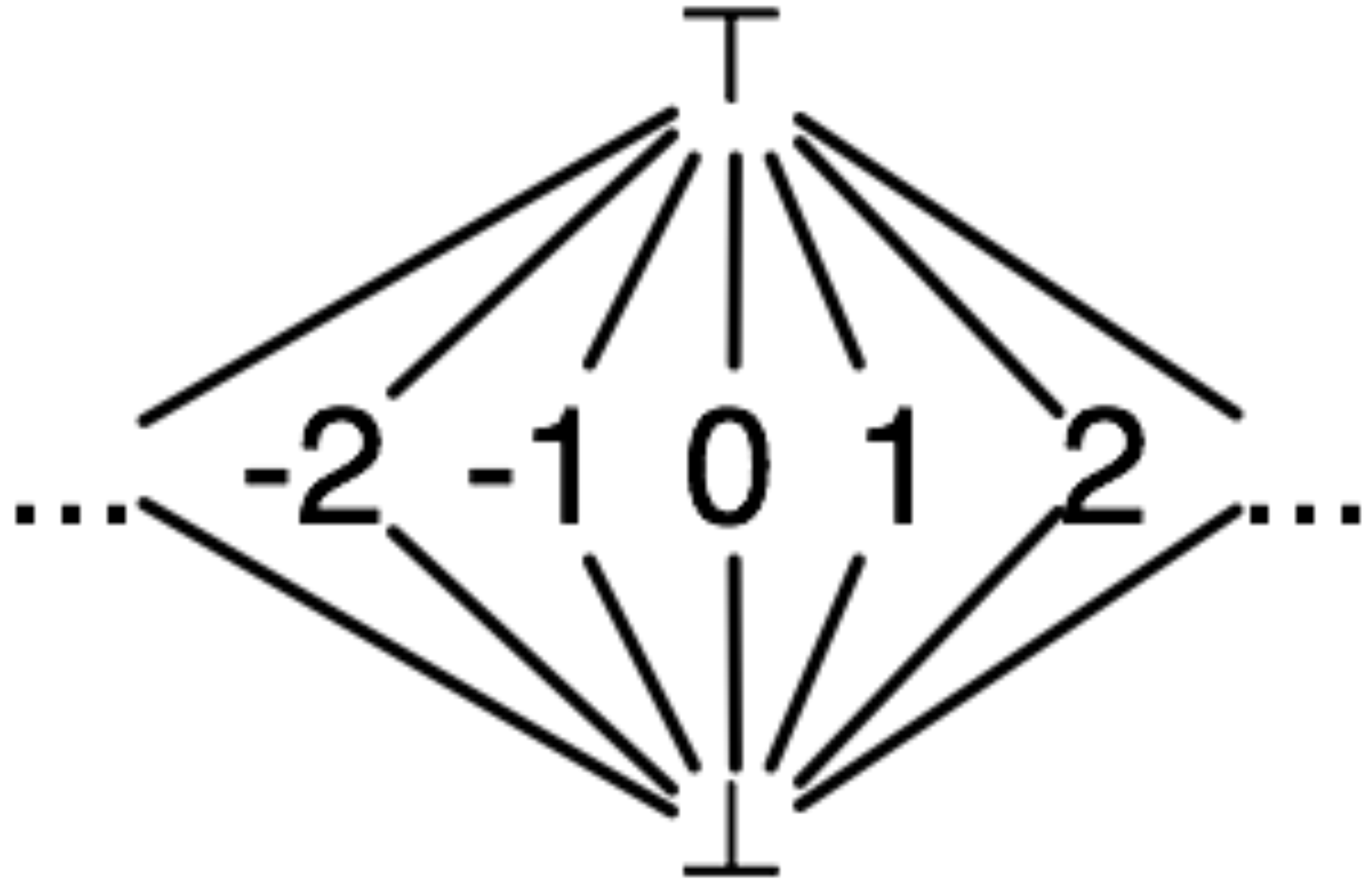
3. $\bot \sqcup * \rightarrow *$

4. $v_1 \sqcup v_2 \rightarrow \top$



how can symbols change?

- Fixpoint algorithm: keep re-executing when a symbol changes
- What happens when a statement executes?
  - If input symbol is “higher” in the lattice, output symbol is “higher” in the lattice
- How can symbols change?
  - $\perp$ → some other symbol the first time the statement is executed
  - some symbol → $\top$ due to merge operations
- Symbols only get larger as symbolic execution continues → symbols can only get as large as $\top$ then stop



next: can we generalize this?